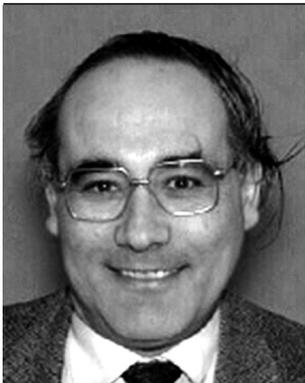


What makes a good maths lecture?

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What is a maths lecture?

By a maths lecture I mean a standard pedagogic lecture given to undergraduates. This distinguishes it from a research talk given to a seminar or conference, and from popular lectures given to school pupils or a general audience, on the one hand, and other forms of undergraduate teaching, such as tutorials, problems classes and revision classes, on the other. I have particularly in mind lectures given to large audiences, say of at least 100 students.

Lectures of this type have the following characteristics:

- a. They present the students with new material.
- b. The lecture is the primary route by which students are exposed to this material.
- c. There may be some interaction with the audience but the agenda is set by the lecturer (or the syllabus), and a main aim of the lecturer will be to transmit a predetermined amount of material.
- d. The purpose of a mathematics lecture is to contribute to the learning of mathematics by the students.

How important are mathematics lectures?

From 2001 to 2009 I led a session on giving mathematics lectures at the MSOR Network Induction Course for New Lecturers. I began these sessions by asking the audience how important lectures were in their own degree courses. There were usually about 30 new lecturers in the audience who had been educated at a wide range of universities in the UK and abroad. Typically on a 4 point scale, 50% of the replies were that lectures were very important, and 40% that they were important.

This confirms my impression that in the 49 years since I first attended a mathematics lecture, very little has changed. Lectures are still the dominant form of teaching provided for mathematics undergraduates, and, despite some technological changes, what happens in the average mathematics lecture is pretty much the same as what happened 40 years ago, if not much earlier. It is not the purpose of this article to discuss whether this continued reliance on traditional lectures is the result of sound judgement or unthinking conservatism.

A paradox

Despite the dominant role of mathematics lectures, the literature on the subject is remarkably sparse. I am aware of very little other than two books by practitioners. Of these, Baumslag [1] is rather disappointing, but that by Krantz [4] is excellent and strongly recommended. There are also some casual remarks by a few others (for example in Chapter 18 of Pota [7]). It is notable that the recent book edited by Kahn and Kyle [3] on effective learning and teaching in mathematics has only a few passing references to lectures. It is also significant that MSORConnections is dominated by articles on the use of computer packages in teaching and has rarely included any mention of lectures.

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However, there are some signs that interest in what makes for a good mathematics lecture is increasing. An interesting view of the role of lectures in teaching mathematics may be found in Fodd [6], which suggests some reasons why the lecture has survived the invention of printing. The recent volume Robinson, Challis and Thomlinson [5] includes many interesting discussions of mathematics teaching in universities, including some illuminating comments about lectures. Bergen [2] reviews some of the literature and using a theoretical framework and observations of a particular lecture puts forward “a systematic triangular model to catch critical quality aspects of a mathematics lecture”. My approach in this article is entirely empirical, that is, anecdotal with some reflection.

Assessing lectures

If the purpose of a mathematics lecture is to contribute to the learning of mathematics by the students, it follows that a good lecture is one that achieves this purpose. But how can this be judged?

We can judge by students’ work on homework problems and by their exam results. Work on homework problems can give some fairly rapid feedback if things are going badly wrong. It is more difficult to interpret examination results since the lecturer usually has the dominant influence on the content and the grading of the examination, and the results don’t just depend on the quality of the lectures. We also hope that our teaching has some long term effects which are not easily measured, even though these may be more

important than the student’s immediate ability to pass an examination. Unfortunately this latter aspect of teaching has become more important as universities have in many cases developed modular systems with examinations immediately at the end of each module. [“Why are you going modular?”, “Well, other universities are doing this”, “Why?”, “For the same reason.”]

Student opinion

You can learn a lot from student surveys of teaching. However, only lecturers who are not sensitive to their audience and who fail to take advantage of informal methods of feedback should be frequently surprised by the results of a formal survey.

My experience (over 25 years of a twice-yearly course survey of first year students) was that when survey forms were filled in during a lecture session, there was a high response rate, but students’ responses were influenced by those they were sitting next to. However, when surveys were completed outside lectures, it was difficult to get a high rate of return. A few survey forms seem to be completed by students still suffering from the effects of the night before. For all these reasons survey results should not be taken too seriously. Unfortunately, these days student surveys have become a fetish [“something regarded with irrational reverence”, is one definition given by my dictionary] and their results are often presented as league tables, without any regard to their statistical significance.

My practice was to ask students how easy they found a lecture course, and whether they found it interesting. To avoid league tables, the results were presented as a table, such as the one below.

	easy	all right	hard
not interesting	3	2	8
all right	7	23	16
interesting	8	29	23

Table 1 – How easy did students find a lecture course

This seems to me to be more informative than condensing the responses into a single number for “interest” and a single number for “difficulty”. This level of subtlety seems to escape most other designers of surveys, but, of course, league tables sell newspapers in a way that matrices do not.

Different students have different needs

As the above table indicates, normally a survey produces a range of opinions. This should not be a surprise. The evidence suggests that different people learn in different ways. Also, a student’s ability to profit from a lecture will depend on their background mathematical knowledge, the pace at which they can absorb mathematics and the level of abstraction they can cope with. These will differ from student to student even when they all have the same A-

level grades. What is a good lecture for one student may be too pedestrian or too advanced for another.

In the light of this, my advice to lecturers is:

- Be sensitive to the audience.
- Try to provide something for everyone.
- Avoid making the average level too high.

I was advised many years ago, that every lecture should contain at least one thing that everyone will understand [“and this is it!”, I would say when a proof ended up with “ $0 + 0 = 0$ ”], and, to impress the audience, at least one thing no-one will understand. I don’t agree with the second half of this advice, but judging from many seminar talks I have sat through, here I am in a minority.

The lack of robust theories of mathematical learning

My understanding is that, although there have been great advances in neuroscience, we do not know enough about how people learn high level mathematics to provide a scientific theory to tell us what makes a good lecture, or what are effective ways to teach mathematics at university level. Indeed, there are still controversies about how best to teach comparatively low level skills, such as long multiplication.

Instead, teaching mathematics remains at the level of craft skill. Some people seem to do it better than others, even though we are unable to articulate recipes that are bound to work well.

From this I draw the following conclusions:

- Avoid dogmatism when pronouncing about teaching mathematics.
- Be flexible and be prepared to experiment, but be wary of very radical changes.
- Respect experience.

However, we should remember that we live in an age during which many craft skills have been completely replaced very quickly by technological advances. In particular, modern technology makes it much easier than before to provide students with lecture notes, and universities are placing more and more emphasis on web-based resources. My impression is that mathematics lecturers are using the web in lots of different ways, but we still have much to learn about good ways to use it, and its impact on lectures. I reserve more discussion of this issue to a possible future article.

The practice of giving mathematics lectures

In the light of this discussion, the best I can do is to give you the benefit of my experience, which is based not only on my own lectures, but also on my reading of several thousand student surveys.

There is lots of very sensible advice in the book by Steven Krantz [4] that I have already recommended. This advice can be summarised as:

Krantz’s maxims

- Prepare well.
- Respect your subject.
- Respect your students.

“The best I can do is to give you the benefit of my experience, which is based not only on my own lectures, but also on my reading of several thousand student surveys.”

In a survey of Leeds University year 1 mathematics students (this was conducted as part of the Student Experiences of Undergraduate Mathematics project, surveying mathematics students from King’s College, University of London and the University of Leeds; Melissa Rodd’s article [6] was one outcome) they were asked to say “how important each of the following is for you when you think about what makes a good lecturer”. Their responses were:

	<i>A lot</i>	<i>Quite a lot</i>	<i>Not much</i>	<i>Not at all</i>
Gives good notes	88%	11%	2%	0%
Gives well organised lectures	75%	23%	2%	1%
Knows the subject	69%	29%	2%	0%
Goes at an appropriate pace	66%	34%	0%	0%
Talks in an interesting way	46%	39%	12%	3%
Is enthusiastic about the subject	43%	47%	8%	2%
Listens to students	27%	49%	22%	2%
Is good fun	26%	45%	29%	1%
Is easy to talk to	22%	38%	35%	4%
Varies the pace of lectures	5%	31%	59%	5%

Table 2 – Survey of Leeds University student on “what makes a good lecturer?”

My summary of students’ preferences, based on all the survey forms I have read is:

Students prefer lecturers who

- Talk to them and not just to the blackboard, and interact with the class.
- Don’t write too fast.
- Have clear large handwriting.
- Provide opportunities to ask questions.
- Give plenty of examples of the theory.

A checklist of tips for new lecturers

I hope that the following list will help new lecturers. Most of these points will be very obvious to anyone with experience

of lecturing, but there are quite a few of them which I wish I had known before I gave my first lecture.

Preparing a course of lectures

1. Check the syllabus. How much flexibility are you allowed?
2. Check what the students can be reasonably be assumed to know at the start.
3. Check the regulations, e.g. how the course is to be assessed, regulations regarding use of calculators or formula sheets in exams.
4. Check the timetable - where and when the lectures take place.
5. Check the facilities in the rooms you will be using.
6. Prepare detailed lecture notes: aim to have a perfect set of lecture notes, but not to need to use them during the lecture (more realistically, to use them very little for the occasional prompt).
7. Think very carefully about the appropriate balance between theory and examples.
8. Think very carefully about the examples you will use.
9. Prepare the homework examples.
10. Prepare any handouts.

Before each lecture

1. Remind yourself where the previous lecture ended, and think about how much you plan to cover in the coming lecture.
2. Check that your lecture notes and any handouts or slides that you plan to use are in order.
3. Rehearse any calculation you plan to do and any difficult proof.
4. Think about how you will use the board, OHP, computer projector, etc.

In the lecture theatre

1. Arrive in good time.
2. Face and talk to the students, establishing eye contact with some of them.
3. Give them a (very brief) reminder of the previous lecture, and a summary what you intend to cover in this lecture.
4. Write clearly on the board.
5. Encourage questions, and respond positively to them.
6. Keep watching the audience. Are they keeping up with writing notes? Do they look baffled or bored? Are they talking to each other?
7. Ask the students questions.

8. End on time.

After the lecture

1. Note the point at which you ended.
2. Write a brief comment for yourself about the lecture, e.g. What changes should you make next time? Was it rushed or did you have plenty of time?

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