

70 Wuste $\begin{pmatrix} a_0 \\ a_1 \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix}$

$$\hat{B} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 7 & -24i \\ 24i & -7 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \lambda \begin{pmatrix} a \\ b \end{pmatrix} \quad \text{for eigenvalues \& eigenvectors.}$$

Eigenvalues given by

$$\det \begin{pmatrix} 7-\lambda & -24i \\ 24i & -7-\lambda \end{pmatrix} = 0 \quad (7-\lambda)(7+\lambda) + 24 \cdot 24 = 0$$

$$\therefore 49 - \lambda^2 + (24)^2 = 0 \quad \lambda^2 = 7^2 + 24^2 = 49 + 576 = 625$$

$$\lambda = \pm 25$$

so eigenvalues are $\lambda = \pm 25$.

eigenvector $\lambda = +25$ $7a - 24ib = 25a \quad \therefore -18a = -24ib \quad \therefore a = -\frac{4ib}{3}$

eigenvector $-\frac{4i}{3}|a_0\rangle + |a_1\rangle$

normalisation $\frac{16}{9} + 1 = \left(\frac{5}{3}\right)^2 \Rightarrow \text{multiply by } \frac{3}{5}$

normalised eigenvector $\underline{-\frac{4i}{5}|a_0\rangle + \frac{3}{5}|a_1\rangle} \quad \underline{25}$

$\lambda = -25$ $7a - 24ib = -25a \quad -24ib = -32a \quad \therefore a = \frac{3ib}{4}$

eigenvector $\frac{3i}{4}|a_0\rangle + |a_1\rangle$

normalisation $\frac{9}{16} + 1 = \left(\frac{5}{4}\right)^2 \quad \therefore \text{normalised eigenvector}$

$\underline{\frac{3i}{5}|a_0\rangle + \frac{4}{5}|a_1\rangle} \quad \underline{-25}$

The state $|a_1\rangle$

After the first measurement we have

$|A\rangle = -\frac{4i}{5}|a_0\rangle + \frac{3}{5}|a_1\rangle$ with probability $| \langle A | a_1 \rangle |^2 = \frac{9}{25}$

or $|B\rangle = \frac{3i}{5}|a_0\rangle + \frac{4}{5}|a_1\rangle$ " " $| \langle B | a_1 \rangle |^2 = \frac{16}{25}$