Q12 Show $\left[\frac{d}{dx}, x^n\right] = nx^{n-1}$ Act on arbitrary f

$$\left(\frac{d}{dx} x^n f\right) - \left(x^n \frac{d}{dx} f\right) = nx^{n-1} f + x^n \frac{d}{dx} f - x^n \frac{d}{dx} f = nx^{n-1} f$$

$$\therefore \left[\frac{d}{dx}, x^n\right] f = nx^{n-1} f \Rightarrow \left[\frac{d}{dx}, x^n\right] = nx^{n-1}$$

$$\left[x\frac{d}{dx}, x\frac{d^2}{dx^2}\right] f = x\frac{d}{dx}\left(x\frac{d^2}{dx^2} f\right) - x\frac{d^2}{dx^2}\left(x\frac{d}{dx} f\right) =$$

$$= x^2 \frac{d^3}{dx^3} f + x\frac{d^2}{dx^2} f - x\frac{d}{dx}\left(\frac{df}{dx} + x\frac{d^2f}{dx^2}\right) =$$

$$= x^2 \frac{d^3f}{dx^3} + x\frac{d^2}{dx^2} f - x\frac{d^2f}{dx^2} - x\frac{d^2f}{dx^2} - x^2\frac{d^3f}{dx^3} = -x\frac{d^2f}{dx^2}$$

$$\text{so} \quad \left[x\frac{d}{dx}, x\frac{d^2}{dx^2}\right] = -x\frac{d^2}{dx^2}$$

$$\left[-i\hbar\frac{d}{dx}, f(x)\right] g = -i\hbar\frac{d}{dx}(fg) + f\left(i\hbar\frac{d}{dx} g\right) = -i\hbar\frac{df}{dx} g$$

$$\text{so} \quad \left[-i\hbar\frac{d}{dx}, f(x)\right] = -i\hbar\frac{df}{dx}.$$