Q13 $(\Psi,\phi) = \int_0^\infty dp \int_0^\infty r\,\Psi^*(r,p)\,\phi(r,p)\,dr$

$\hat{A} = a\left(p\frac{\partial}{\partial p} - br\frac{\partial}{\partial r}\right)$ is self-adjoint if

$$\langle\Psi|A|\phi\rangle = \langle\phi|A|\Psi\rangle^*$$

For this to be true we must have

$$a\int_0^\infty dp\int_0^\infty dr\, r\Psi^*\left(p\frac{\partial\phi}{\partial p} - br\frac{\partial\phi}{\partial r}\right) = \left[a\int_0^\infty dp\int_0^\infty dr\,\phi^* r\left(p\frac{\partial\Psi}{\partial p} - br\frac{\partial\Psi}{\partial r}\right)\right]^*$$

Clearly if scalar product defined as above

$\Psi,\phi(r,p) \xrightarrow[p\to\infty]{} 0$ and $\Psi,\phi \xrightarrow[r\to\infty]{} 0$ fast enough.

So integrate by parts.

$$a\int_0^\infty dp\int_0^\infty dr\, r\Psi^*\left(p\frac{\partial\phi}{\partial p} - br\frac{\partial\phi}{\partial r}\right) = a\int_0^\infty dr\, r\,\Psi^* p\,\phi\Big|_0^\infty - ab\int_0^\infty dp\, r^2\Psi^*\phi\Big|_0^\infty$$

$$-a\int_0^\infty dp\int_0^\infty dr\, r\Psi^*\phi - a\int_0^\infty dp\int_0^\infty dr\, r\,p\frac{\partial\Psi^*}{\partial p}\phi + ab\cdot 2\int_0^\infty dp\int_0^\infty dr\, r\Psi^*\phi$$

$+ab\int_0^\infty dp\int_0^\infty dr\, r^2\frac{\partial\Psi^*}{\partial r}\phi$. The first two terms vanish due to *

Terms $\int_0^\infty dp\int_0^\infty dr\, r\Psi^*\phi - 2b\int_0^\infty dp\int_0^\infty dr\, r\Psi^*\phi$ cancel if $b = \frac{1}{2}$

and we have hermiticity (self-adjointness)

if $a = ik$ where k is real.

$\therefore$ $a = ik$ k real, $b = \frac{1}{2}$.