Quantum Mechanics - Solutions

Q15

$$\hat{R}|1\rangle = \alpha|1\rangle \qquad \hat{R}|2\rangle = \alpha|2\rangle$$

$$\langle 1|\hat{Q}|2\rangle = -\langle 1|\hat{R}\hat{Q}\hat{R}|2\rangle = -\alpha^2\langle 1|\hat{Q}|2\rangle$$

but for self adjoint operator α real

$$\langle 1|\hat{Q}|2\rangle(1+\alpha^2) = 0 \quad \Rightarrow \quad \langle 1|\hat{Q}|2\rangle = 0$$

Q21 Calculate eigenvalues of $\begin{pmatrix}1 & 1\\0 & 1\end{pmatrix}$

$$\begin{pmatrix}1 & 1\\0 & 1\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} = \lambda\begin{pmatrix}x\\y\end{pmatrix} \quad \therefore \begin{vmatrix}1-\lambda & 1\\0 & 1-\lambda\end{vmatrix} = 0 \quad \lambda = +1.$$

so eigenvalue is 1 only 1.

eigenvector $\quad x+y = x \quad y = y \qquad \therefore y = 0$, x arbitrary.

so $\begin{pmatrix}1 & 1\\0 & 1\end{pmatrix}$ has one eigenvector $\begin{pmatrix}1\\0\end{pmatrix}$ with eigenvalue 1

Eigenvectors are <u>not complete</u> as state $\begin{pmatrix}0\\1\end{pmatrix}$ cannot be expressed in terms of them.

Clear as $\begin{pmatrix}1 & 1\\0 & 1\end{pmatrix}$ is not self adjoint.

So this matrix cannot represent an observable in QM.