Quantum Mechanics - Solutions

Q39 continued

so normalised $|f\rangle = \exp\left(\frac{-f^2}{2\hbar\omega}\right)\left(\sum_{k=0}^{\infty} \frac{f^k}{(\hbar\omega)^{k/2}\sqrt{k!}}|k\rangle\right)$

and
$$\langle g|f\rangle = \exp\left(\frac{-f^2-g^2}{2\hbar\omega}\right)\sum_{k=0}^{\infty}\frac{f^k g^k}{(\hbar\omega)^k k!} = \exp\left[\frac{-(f-g)^2}{2\hbar\omega}\right].$$

$$\hat{C}(f) = ? \qquad |k\rangle = \frac{1}{\sqrt{k!(\hbar\omega)^k}}(\hat{a})^k|0\rangle$$

so $|f\rangle = \exp\left(\frac{-f^2}{2\hbar\omega}\right)\sum_{k=0}^{\infty}\frac{f^k(\hat{a})^k}{k!(\hbar\omega)^k}|0\rangle = \exp\left(\frac{-f^2}{2\hbar\omega}+\frac{f\hat{a}}{\hbar\omega}\right)|0\rangle$

so $\hat{C}(f) = \exp\left(\frac{-f^2}{2\hbar\omega}\right)\exp\left(\frac{f\hat{a}}{\hbar\omega}\right)$

We cannot perform a similar construction of eigenstates of $\hat{a}$ as when we try it

$$|f\rangle = d_0|0\rangle + d_1|1\rangle + d_2|2\rangle + \ldots$$

$$a|f\rangle = d_0\sqrt{\hbar\omega}\,|1\rangle + d_1\sqrt{2\hbar\omega}\,|2\rangle + \ldots$$

but clearly $|0\rangle$ state is not in $a|f\rangle$ ..
$\Rightarrow$ all coefficients vanish.

Resolution of $\hat{I}$ is quite complicated as $|f\rangle$ states are not orthogonal.