41 $\quad \hat{H} = \frac{1}{2}\hbar\begin{pmatrix} 3 & i \\ -i & 3 \end{pmatrix} \qquad \Psi(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

what are eigenvalues and eigenstates of $\hat{H}$

Eigenvalues $\quad \det\begin{vmatrix} 3-\lambda & i \\ -i & 3-\lambda \end{vmatrix} = 0 \quad \therefore \lambda = 2,\ \lambda = 4$

so eigenvalues are $\hbar,\ 2\hbar \qquad E_0 = \hbar,\quad E_1 = 2\hbar$

and eigenvectors

E_0: $\quad \frac{1}{2}\hbar\begin{pmatrix} 3 & i \\ -i & 3 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \hbar\begin{pmatrix} a \\ b \end{pmatrix} \quad \therefore \frac{3}{2}a + \frac{i}{2}b = a$

$\therefore a + ib = 0$

eigenvector $\begin{pmatrix} -i \\ 1 \end{pmatrix}$ $\Leftrightarrow$ corresponding to eigenvalue E_0

other eigenvector

$\frac{1}{2}\hbar\begin{pmatrix} 3 & i \\ -i & 3 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = 2\hbar\begin{pmatrix} a \\ b \end{pmatrix} \quad \therefore \frac{3}{2}a + \frac{i}{2}b = 2a \quad \therefore a = ib$

$\begin{pmatrix} i \\ 1 \end{pmatrix}$ eigenvector

so initially

$\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \alpha\begin{pmatrix} -i \\ 1 \end{pmatrix} + \beta\begin{pmatrix} i \\ 1 \end{pmatrix} \quad \therefore \begin{matrix} \alpha + \beta = 0 \\ i(\beta - \alpha) = 1 \end{matrix} \ \Bigg| \Rightarrow \begin{matrix} \alpha = \frac{i}{2} \\ \beta = -\frac{i}{2} \end{matrix}$

so $\Psi(0) = \left(\frac{i}{2}\right)\left[\begin{pmatrix} -i \\ 1 \end{pmatrix} - \begin{pmatrix} i \\ 1 \end{pmatrix}\right]$

$$\Psi(t) = e^{-\frac{i\hat{H}t}{\hbar}}\Psi(0) = \frac{i}{2}\left[\begin{pmatrix} -i \\ 1 \end{pmatrix}e^{-it} - \begin{pmatrix} i \\ 1 \end{pmatrix}e^{-2it}\right] =$$

$$= \frac{i}{2}\begin{bmatrix} -i\left(e^{-it} + e^{-2it}\right) \\ e^{-it} - e^{-2it} \end{bmatrix} = \frac{1}{2}\begin{pmatrix} e^{-it} + e^{-2it} \\ i\left(e^{-it} - e^{-2it}\right) \end{pmatrix} = \begin{pmatrix} \frac{1}{2}\left(e^{-it} + e^{-2i\ldots}\right. \\ \frac{i}{2}\left(e^{-it} - e^{-2i\ldots}\right. \end{pmatrix}$$