Quantum Mechanics – Solutions

41 continued

$Q = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ eigenvalues are $\det \begin{vmatrix} \lambda & 1 \\ 1 & \lambda \end{vmatrix} = 0 \quad \lambda = \pm 1$

eigenvectors are given by

$\lambda = +1 \quad \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} \quad \therefore b = a \quad \therefore \begin{pmatrix} 1 \\ 1 \end{pmatrix}\frac{1}{\sqrt{2}}$

$\lambda = -1 \quad \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = -\begin{pmatrix} a \\ b \end{pmatrix} \quad b = -a \quad \begin{pmatrix} 1 \\ -1 \end{pmatrix}\frac{1}{\sqrt{2}}$ normalized

So measurement of Q will give either $+1$ or -1

probabilities of obtaining $+1$ or -1 are given by $|a|^2$ and $|b|^2$ respectively

$$\psi(t) = a\, e_1 + b\, e_2$$

where $e_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}\frac{1}{\sqrt{2}}, \quad e_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}\frac{1}{\sqrt{2}}$

so $|a|^2 = |e_1 \cdot \psi|^2 = \frac{1}{2}\left| \frac{1}{2}\left(e^{-it} + e^{-2it}\right) + \frac{i}{2}\left(e^{-it} - e^{-2it}\right)\right|^2 =$

as e_1, e_2 are normalized

$$= \frac{1}{4 \cdot 2}\left| e^{-it}(1+i) + e^{-2it}(1-i)\right|^2 = \frac{1}{8}\left[|1+i|^2 + |1-i|^2 + (i+1)^2 e^{it} + (1-i)^2 e^{-it}\right]$$

$$= \frac{1}{8}\left[4 + 2ie^{it} - 2ie^{-it}\right] = \frac{1}{2}\left[1 - \sin t\right]$$

so probability of finding $+1$ is $\frac{1}{2}(1 - \sin t)$

-1 is $\frac{1}{2}(1 + \sin t)$