41 continued (end).

$$\langle Q\rangle = \sum \text{value} * \text{probability} = 1\cdot\tfrac{1}{2}(1-\sin t) + (-1)\tfrac{1}{2}(1+\sin t)$$
$$= -\sin t.$$

On the other hand

$$[\hat H, \hat Q] = \tfrac{1}{2}\hbar\left[\begin{pmatrix}3 & i\\ -i & 3\end{pmatrix}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} - \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}3 & i\\ -i & 3\end{pmatrix}\right] = \tfrac{1}{2}\hbar\left(\begin{pmatrix}i & 3\\ 3 & -i\end{pmatrix} - \begin{pmatrix}-i & 3\\ 3 & i\end{pmatrix}\right)$$
$$= i\hbar\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

Next we evaluate $\langle[\hat H, \hat Q]\rangle$ on our state $\psi(t)$.

Put $\psi(t) = \begin{pmatrix}a(t)\\ b(t)\end{pmatrix}$ $\therefore$ $\langle[\hat H, \hat Q]\rangle = \left(a^*(t), b^*(t)\right) i\hbar \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}a(t)\\ b(t)\end{pmatrix}$

$$= \left[a^*(t), b^*(t)\right] i\hbar\begin{pmatrix}a(t)\\ -b(t)\end{pmatrix} = i\hbar\left(|a(t)|^2 - |b|^2\right)$$

so $|a(t)|^2 - |b(t)|^2 = \frac{1}{4}\left[\left|e^{-it} + e^{-2it}\right|^2 - \left|e^{-it} - e^{-2it}\right|^2\right] =$

$$= \tfrac{1}{4}\left(2e^{it} + 2e^{-it}\right) = \cos t$$

The equation to be satisfied is

$$\frac{d}{dt}\langle \hat A\rangle = \frac{1}{i\hbar}\langle[\hat A, \hat H]\rangle$$

in our case $\langle \hat A\rangle = \langle \hat Q\rangle = -\sin t$

$\langle[\hat A, \hat H]\rangle = -\langle[\hat H, \hat Q]\rangle = -\cos t\,(i\hbar)$ so OK.