Math 164-1: Optimization – Homework 3

Due: October 16, 2015

Exercise 1. *Exercises 6.13, 6.14 and 6.26 from the book of Chong and Zak.*

(i) (Exercise 6.13) Consider the problem

$$\min_{x \in \Omega} f(x),$$

where $f : \mathbb{R}^2 \to \mathbb{R}$ is given by $f(x) = -3x_1$ with $x = [x_1, x_2]^T$, and $\Omega = \{x = [x_1, x_2]^T : x_1 + x_2^2 \leq 2\}$. Answer each of the following questions, showing complete justification.

a. Does the point $x^* = [2, 0]^T$ satisfy the first-order necessary condition?

b. Does the point $x^* = [2, 0]^T$ satisfy the second-order necessary condition?

c. Is the point $x^* = [2, 0]^T$ a local minimizer?

(ii) (Exercise 6.14) Consider the problem

$$\min_{x \in \Omega} f(x),$$

where $\Omega = \{x \in \mathbb{R}^2 : x_1^2 + x_2^2 \geq 1\}$ and $f(x) = x_2$.

(a) Find all point(s) satisfying the first order optimality condition.

(b) Which of the point(s) in part (a) satisfy the second order optimality condition?

(c) Which of the point(s) in part (a) are local minimizers?

(iii) (Exercise 6.26) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a $C^1(\mathbb{R}^2)$ function. Consider the problem

$$\min_{x_1, x_2 \geq 0} f(x),$$

where $x = [x_1, x_2]^T$. Suppose that $\nabla f(0) \neq 0$, and

$$\frac{\partial f}{\partial x_1}(0) \leq 0, \frac{\partial f}{\partial x_2}(0) \leq 0.$$

Show that 0 cannot be a minimizer for this problem.

Exercise 2. *Exercises 7.1 and 7.4 from the book of Chong and Zak.*

(i) (Exercise 7.1) Suppose that we have a unimodal function over the interval $[5, 8]$. Give an example of a desired final uncertainty range where the golden section method requires at least four iterations, whereas the Fibonacci method requires only three. You may choose an arbitrarily small value of ε for the Fibonacci method.

(ii) (Exercise 7.4) Suppose that $\rho_1, \ldots, \rho_N$ are the values used in the Fibonacci search method. Show that for each $k = 1, \ldots, N$, $0 \leq \rho_k \leq 1/2$, and for each $k = 1, \ldots, N-1$,

$$\rho_{k+1} = 1 - \frac{\rho_k}{1 - \rho_k}.$$

Exercise 3. *Exercises 7.2 from the book of Chong and Zak.*

(i) (Exercise 7.2) Let $f(x) = x^2 + 4\cos x$, $x \in \mathbb{R}$. We wish to find the minimizer x^* of f over the interval $[1, 2]$. (*Calculator users:* Note that in $\cos x$, the argument x is in radians.)

(a) Plot $f(x)$ versus x over the interval $[1, 2]$.

(b) Use the golden section method to locate x^* to within an uncertainty of 0.2. Display all intermediate steps using a table:

Iteration k	a_k	b_k	$f(a_k)$	$f(b_k)$	New uncertainty interval
1	?	?	?	?	$[?, ?]$
2	?	?	?	?	$[?, ?]$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

(c) Repeat part b. using the Fibonacci method, with $\varepsilon = 0.05$. Display all intermediate steps using a table:

Iteration k	ρ_k	a_k	b_k	$f(a_k)$	$f(b_k)$	New uncertainty interval
1	?	?	?	?	?	$[?, ?]$
2	?	?	?	?	?	$[?, ?]$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

(d) Apply Newton's method, using the same number of iterations as in part (b), with $x^{(0)} = 1$.

Exercise 4. *Exercises 7.5 and 7.6 from the book of Chong and Zak.*

(i) (Exercise 7.5) Show that if $F_0, F_1, \ldots$ is the Fibonacci sequence, then for each $k = 2, 3, \ldots,$

$$F_{k-2}F_{k+1} - F_{k-1}F_k = (-1)^k.$$

(ii) (Exercise 7.6) Show that the Fibonacci sequence can be calculated using the formula

$$F_n = \frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^{n+1} - \left(\frac{1-\sqrt{5}}{2}\right)^{n+1}\right).$$