Math 164-1: Optimization – Homework 7

Due: November 13, 2015

Exercise 1.

Exercises 10.6 and 10.7 from the book of Chong and Zak.

Exercise 2.

Exercises 11.1 and 11.2 from the book of Chong and Zak.

Exercise 3.

Exercises 11.3, 11.5 and ~~11.8~~ 11.7 from the book of Chong and Zak.

Exercise 4.

(1) Exercises 12.18 from the book of Chong and Zak. Hint: since this is an optimization problem with constraint, one should compute first the feasible directions at a point in $\mathcal{R}(A)$. Then you may use some theorem from Chapter 3.

(2) Let $A \in \mathbb{R}^{m\times n}$, $n, m \in \mathbb{N}$. Show that AA^T and A^TA are symmetric positive semidefinite matrices. What is the condition for A, that ensures that AA^T and A^TA are positive definite?

(3) Let

$$A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \\ 2 & 1 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}.$$

We are interested in solving numerically the system $Ax = b$, where $x \in \mathbb{R}^2$. Use the conjugate gradient algorithm to solve this system in two steps, by minimizing the function $x \mapsto \frac{1}{2}\|Ax - b\|^2$. Describe the algorithm and compute all the necessary terms at each step. Check that the candidate you have found after 2 steps is indeed a solution.