Math 164: Optimization – Homework 3

Due: October 14, 2016

Exercise 1 (Alternative sufficient optimality condition on networks)**.**

(1) Suppose that $\Omega \subset \mathbb{R}^d$ is a closed *network*, meaning that it is the union of line segments in $\mathbb{R}^d$ such that at any intersection (junction) meet only finitely many line segments. A good example would be a road map of a country, where all the roads are straight.

We consider moreover $f : \mathbb{R}^d \to \mathbb{R}$ a C^1 function. Show that if $x^* \in \Omega$ satisfies the condition

$$\nabla f(x^*) \cdot e > 0,$$

for all feasible direction $e \in \mathbb{R}^d$ at x^* (and we suppose that there exists at least one feasible direction at x^*) then x^* is a strict local minimizer of f on Ω.

Hint: use the definition of strict local minimizer. Use the fact that any point x in a small neighborhood of x^* can be represented as $x^* + te$ for some $t > 0$ and feasible direction e at x^* and you can have only finitely many directions like this.

(2) Give an example of $\Omega \subset \mathbb{R}^2$ nonconvex set for which the implication in (1) does not hold true.

(3) Define a set $\Omega \in \mathbb{R}^2$ and a point $x^* \in \Omega$ such that there exists precisely 3 feasible directions $e_1, e_2, e_2 \in \mathbb{R}^2$ at x^* (in this context we take two feasible directions to be the same, if one is a positive scalar multiple of the other)! We know that for a C^1 function $f : \mathbb{R}^2 \to \mathbb{R}$ one has that $\nabla f(x^*) \cdot e_1 > 0$ and $\nabla f(x^*) \cdot e_2 = 0, \nabla f(x^*) \cdot e_3 = 0$. Can we say that x^* is a strict local minimizer of f on Ω? Justify your answer.

Exercise 2.

Exercises 20.2 and 20.3 from the book of Chong and Zak.

Exercise 3.

Exercises 20.4 and 20.5 from the book of Chong and Zak.

Exercise 4.

Exercises 20.6, 20.12 and 20.13 from the book of Chong and Zak.