Math 164-1: Optimization – Homework 3

Due: April 15, 2016

Exercise 1. *Exercises 6.13, 6.14 and 6.26 from the book of Chong and Zak.*

(i) (Exercise 6.13) Consider the problem

$$\min_{x\in\Omega} f(x),$$

where $f : \mathbb{R}^2 \to \mathbb{R}$ is given by $f(x) = -3x_1$ with $x = [x_1, x_2]^T$, and $\Omega = \{x = [x_1, x_2]^T : x_1 + x_2^2 \leq 2\}$. Answer each of the following questions, showing complete justification.

(a) Does the point $x^* = [2, 0]^T$ satisfy the first-order necessary condition?

(b) Does the point $x^* = [2, 0]^T$ satisfy the second-order necessary condition?

(c) Is the point $x^* = [2, 0]^T$ a local minimizer?

(ii) (Exercise 6.14) Consider the problem

$$\min_{x\in\Omega} f(x),$$

where $\Omega = \{x \in \mathbb{R}^2 : x_1^2 + x_2^2 \geq 1\}$ and $f(x) = x_2$.

(a) Find all point(s) satisfying the first order optimality condition.

(b) Which of the point(s) in part (a) satisfy the second order optimality condition?

(c) Which of the point(s) in part (a) are local minimizers?

(iii) (Exercise 6.26) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a $C^1(\mathbb{R}^2)$ function. Consider the problem

$$\min_{x_1, x_2 \geq 0} f(x),$$

where $x = [x_1, x_2]^T$. Suppose that $\nabla f(0) \neq 0$, and

$$\frac{\partial f}{\partial x_1}(0) \leq 0, \frac{\partial f}{\partial x_2}(0) \leq 0.$$

Show that 0 cannot be a minimizer for this problem.

Exercise 2 (Alternative optimality conditions)**.**

(1) Suppose that $\Omega \subset \mathbb{R}^d$ is a closed bounded convex set and $f : \mathbb{R}^d \to \mathbb{R}$ is a C^1 function. Show that if $x^* \in \Omega$ satisfies the condition

$$\nabla f(x^*) \cdot e > 0,$$

for all feasible direction $e \in \mathbb{R}^d$ at x^* (and we suppose that there exists at least one feasible direction at x^*) then x^* is a strict local minimizer of f on Ω.

(2) Define a set $\Omega \in \mathbb{R}^2$ and a point $x^* \in \Omega$ such that there exists precisely 3 feasible directions $e_1, e_2, e_2 \in \mathbb{R}^2$ at x^*! We know that for a C^1 function $f : \mathbb{R}^2 \to \mathbb{R}$ one has that $\nabla f(x^*) \cdot e_1 > 0$ and $\nabla f(x^*) \cdot e_2 = 0, \nabla f(x^*) \cdot e_3 = 0$. Can we say that x^* is a strict local minimizer of f on Ω? Justify your answer.

Exercise 3.

Exercises 20.2 and 20.3 from the book of Chong and Zak.

Exercise 4.

Exercises 20.4. and 20.5 from the book of Chong and Zak.