Math 32B-1: Calculus of Several Variables – Homework 1

Due: January 8, 2016

Exercise 1 (Based on Rogawski-Adams)**.**

(1) Let $D = [0,1] \times [0,1]$. Estimate $\int\int_D (x+2y)\,\mathrm{d}A$ by computing two different Riemann sums, each with at least six rectangles.

(2) Evaluate $\int\int_D 14\,\mathrm{d}A$, where $D = [2,5] \times [4,7]$.

(3) Evaluate $\int\int_D (15-3x)\,\mathrm{d}A$, where $D = [0,5] \times [0,3]$, and sketch the corresponding solid region.

(4) Evaluate $\int\int_D (-7)\,\mathrm{d}A$, where $D = [2,5] \times [4,7]$.

Exercise 2 (Based on Rogawski-Adams)**.**

(1) $\int\int_D x^3\,\mathrm{d}A$, $D = [-4,4] \times [0,5]$.

(2) $\int\int_D (2+x^2y)\,\mathrm{d}A$, $D = [0,1] \times [-2,2]$.

(3) $\displaystyle\int_1^3 \int_0^2 x^3 y\,\mathrm{d}y\,\mathrm{d}x$

(4) $\displaystyle\int_{-1}^1 \int_0^\pi x^2 \cos y\,\mathrm{d}x\,\mathrm{d}y$

Hint: you can use symmetry to evaluate the first two integrals.

Exercise 3 (Based on Rogawski-Adams)**.**

Compute the following integrals:

(1) $\displaystyle\int_1^2 \int_4^8 e^{3x-y}\,\mathrm{d}y\,\mathrm{d}x$.

(2) $\displaystyle\int\int_D \frac{x}{y+3}\,\mathrm{d}A$, where $D = [0,2] \times [0,4]$.

(3) $\displaystyle\int_0^1 \int_0^2 xe^{xy}\,\mathrm{d}x\,\mathrm{d}y$. *Hint:* change the order of integration.

Exercise 4 (From Rogawski-Adams)**.**

Prove the following extension of the Fundamental Theorem of Calculus to two variables: if $\dfrac{\partial^2 F}{\partial x \partial y}(x,y) = f(x,y)$ for any $(x,y) \in D$, then

$$\int\int_D f(x,y)\,\mathrm{d}A = F(b,d) - F(a,d) - F(b,c) + F(a,c),$$

where $D = [a,b] \times [c,d]$.