Math 32B-1: Calculus of Several Variables – Homework 5

Due: February 5 , 2016

Exercise 1 (From Rogawski-Adams).

(1) Use spherical coordinates to find the volume of a sphere of radius 2 from which a central cylinder of radius 1 has been removed.

For (2) and (3) use spherical coordinates to calculate the triple integral of $f(x, y, z)$ over the given region.

(2) $f(x, y, z) = (x^2 + y^2 + z^2)^{-3/2}$; $2 \leq x^2 + y^2 + z^2 \leq 4$

(3) $f(x, y, z) = 1$; $x^2 + y^2 + z^2 \leq 4$, $z \geq \sqrt{x^2 + y^2}$

Exercise 2 (From Rogawski-Adams).

Use cylindrical coordinates to calculate the triple integral for the given function and region

(1) $f(x, y, z) = z\sqrt{x^2 + y^2}$; $x^2 + y^2 \leq z \leq 8 - (x^2 + y^2)$.

(2) $f(x, y, z) = z$; $0 \leq z \leq x^2 + y^2 \leq 9$

Express the triple integral in cylindrical coordinates

(3) $\displaystyle\int_0^1 \int_{y=-\sqrt{1-x^2}}^{y=\sqrt{1-x^2}} \int_{z=0}^{4} f(x, y, z)\, \mathrm{d}z\, \mathrm{d}y\, \mathrm{d}x$

(4) $\displaystyle\int_0^2 \int_{y=0}^{y=\sqrt{2x-x^2}} \int_{z=0}^{\sqrt{x^2+y^2}} f(x, y, z)\, \mathrm{d}z\, \mathrm{d}y\, \mathrm{d}x$

Exercise 3 (From Rogawski-Adams).

(1) Find the total population within a 4-km radius of the city center (located at the origin) assuming a population density of $\delta(x, y) = 2000(x^2 + y^2)^{-0.2}$ people per square kilometer.

(2) Find the total mass of the solid region W defined by $x \geq 0$, $y \geq 0$, $x^2 + y^2 \leq 4$ and $x \leq z \leq 32 - x$ (in centimeters) assuming a density of $\delta(x, y, z) = 6y\ g/cm^3$.

(3) Find the centroid of the region bounded by $y^2 = x + 4$ and $x = 4$ assuming the density $\delta(x, y) = 1$.

Exercise 4 (From Rogawski-Adams).

(1) Find the center of mass of the region bounded by $y^2 = x+4$ and $x = 0$ with the density $\delta(x, y) = |y|$.

(2) Find the center of mass of the semicircle $x^2 + y^2 \leq R^2$ and $y \geq 0$ assuming the density $\delta(x, y) = y$.

(3) Find the center of mass of the cylinder of radius 2 and height 4 and mass density e^{-z}, where z is the height above the base.