Math 32B-1: Calculus of Several Variables – Homework 6

Due: February 12 , 2016

Exercise 1 (From Rogawski-Adams)**.**

(1) Sketch an example of a non-constant vector field in the plane in which each vector is parallel to the vector $(1;1)$.

(2) Give an example of a potential function for the vector field $F(x,y,z) = (yz, xz, xy)$ other than $f(x,y,z) = xyz$.

(3) Sketch the following vector fields in the plane by drawing the vectors attached to points with integer coordinates in the rectangle $-4 \leq x \leq 4$ and $-3 \leq y \leq 3$. $F_1(x,y) = (1/2;1/2)$; $F_2(x,y) = \left(-\dfrac{y}{\sqrt{x^2+y^2}}; -\dfrac{x}{\sqrt{x^2+y^2}}\right)$; $F_3(x,y) = (2x+2;y)$.

(4) Compute the divergence and the curl of the following vector fields: $F_1(x,y) = (y/x; y/z; z/x)$; $F_2(x,y,z) = (e^{xy}; \cos(xy); \sin(z))$; $F_3(x,y,z) = (z^2; x^2; y^2)$.

Exercise 2 (From Rogawski-Adams)**.**

(1) The velocity vectors in km/h for the wind speed of a tornado near the ground are given by the vector field $F(x,y) = \left(-\frac{y}{e^{(x^2-y^2-1)^2}}, \frac{x}{e^{(x^2-y^2-1)^2}}\right)$. Determine the coordinates of those points where the wind speed is the highest.

(2) Find a potential function for the vector field $F(x,y) = (x,0)$; and prove that $G(x,y) = (y,0)$ is not conservative.

(3) Prove that $\text{curl}(fF) = f\text{curl}(F) + (\nabla f) \times F$, where f is a scalar function and F is a vector field.

(4) Show that $\text{div}(fF) = \nabla f \cdot F + f\text{div}(F)$, where f is a scalar function and F is a vector field.

Exercise 3 (From Rogawski-Adams)**.**

Compute the line integrals $\displaystyle\int_C f \, \mathrm{d}s$ of the following functions on the specified curves

(1) $f(x,y) = \frac{y^3}{x^7}$; and C is given by $y = (1/4)x^4$ for $1 \leq x \leq 2$.

(2) $f(x,y,z) = 3x - 2y + z$; $r(t) = (2+t, 2-t, 2t)$ where $-2 \leq t \leq 1$.

(3) $f(x,y,z) = x^2 z$; $r(t) = (e^t, \sqrt{2}t, e^{-t})$ where $0 \leq t \leq 1$.

Exercise 4 (From Rogawski-Adams)**.**

Compute the line integrals $\displaystyle\int_C F \cdot \mathrm{d}r$ of the following vector fields on the specified curves

(1) $F(x,y,z) = (z^2, x, y)$ and C is given by $r(t) = (3+5t^2; 3-t^2, t)$ for $0 \leq t \leq 2$.

(2) $F(x,y,z) = (xy, 2, z^3)$ on the Helix given by $r(t) = (\cos(t), \sin(t), t)$ for $t \in [0, \pi]$.

(3) $F(x,y) = (4,y)$ on the quarter of the circle $x^2 + y^2 = 1$ with $x \leq 0, y \geq 0$, oriented clockwise.