Math 32B-1: Calculus of Several Variables – Homework 7

Due: February 19 , 2016

Exercise 1 (From Rogawski-Adams)**.**

Compute $\int_C F \cdot \mathrm{d}r$ for the following oriented curves:

(1) $F(x,y) = (e^{y-x}, e^{2x})$, piecewise linear path from $(1,1)$ to $(2,2)$ to $(0,2)$.

(2) $F(x,y) = \left(\dfrac{-y}{(x^2+y^2)^2}, \dfrac{x}{(x^2+y^2)^2}\right)$, circle of radius R with center at the origin oriented counter-clockwise.

(3) $F(x,y,z) = (z^3, yz, x)$, quarter of the circle of radius 2 in the yz-plane with center at the origin where $y \geq 0$ and $z \geq 0$, oriented clockwise when viewed from the positive x-axis.

(3) $F(x,y,z) = (e^z, e^{x-y}, e^y)$, the closed path between $(0,0,6)$, $(0,4,0)$ and $(2,0,0)$.

Exercise 2 (From Rogawski-Adams)**.**

(1) Calculate the total mass of a metal tube in the helical shape $r(t) = (\cos t, \sin t, t^2)$ (distance in centimeters) for $0 \leq t \leq 2\pi$ if the mass density is $\delta(x,y,z) = \sqrt{z}$ g/cm.

(2) Calculate the work done by the field $F(x,y,z) = (x,y,z)$ when the object moves along $r = (\cos t, \sin t, t)$ for $0 \leq t \leq 3\pi$.

(3) Calculate the work done by the field $F(x,y,z) = (e^x, e^y, xyz)$ when the object moves along $r = (t^2, t, t/2)$ for $0 \leq t \leq 1$.

(4) Let $F(x,y) = (y,x)$. Prove that if C is any path from (a,b) to (c,d), then $\int_C F \cdot \mathrm{d}r = cd - ab$.

Exercise 3 (From Rogawski-Adams)**.**

(1) Let $F(x,y,z) = (x^{-1}z, y^{-1}z, \ln(xy))$.

   (a) Verify that $F = \nabla f$, where $f(x,y,z) = z\ln(xy)$.

   (b) Evaluate $\int_C F \cdot \mathrm{d}r$, where $r(t) = (e^t, e^{2t}, t^2)$ for $1 \leq t \leq 3$.

   (c) Evaluate $\int_C F \cdot \mathrm{d}r$, for any path from $P = (1/2, 4, 2)$ to $Q = (2,2,3)$ contained in the region $x > 0$, $y > 0$.

   (d) Why is it necessary to specify that the path lies in the region where x and y are positive?

(2) Verify that $F = \nabla f$, where $F(x,y) = (\cos y, -x\sin y)$, $f(x,y) = x\cos y$ and evaluate the line integral of F over the upper half of the unit circle centered at the origin, oriented counterclockwise.

Exercise 4 (From Rogawski-Adams)**.**

Find a potential function for F or determine that F is not conservative.

(1) $F_1(x,y,z) = (y,x,z^3)$, $F_2(x,y,z) = (\cos z, 2y, -x\sin z)$, $F_3(x,y,z) = (e^x(z+1), -\cos y, e^x)$, $F_4(x,y,z) = (yze^{xy}, xze^{xy} - z, e^{xy} - y)$.

(2) Let $F(x,y) = \left(\dfrac{1}{x}, \dfrac{-1}{y}\right)$. Calculate the work against F required to move from $(1,1)$ to $(3,4)$ along any path in the first quadrant.