Math 32B-1: Calculus of Several Variables – Homework 8

Due: February 26, 2016

Exercise 1.

Review the exercises from Homework 5, 6, 7, this is a first source of preparation for the midterm.

Exercise 2 (From Rogawski-Adams)**.**

(1) What is the outward-pointing unit normal to the sphere of radius 3 centered at $P = (2, 2, 1)$?.

(2) Show that $G(r, \theta) = (r\cos\theta, r\sin\theta, 1 - r^2))$ parametrizes the paraboloid $z = 1 - x^2 - y^2$. Describe the grid curves of this parametrization.

(3) Let $S = G(D)$, where $D = \{(u, v) : u^2 + v^2 \leq 1; u \geq 0, v \geq 0\}$ and $G(u, v) = (2u + 1, u - v, 3u + v)$. Compute the surface area of S.

Exercise 3 (Based on Rogawski-Adams)**.**

Compute T_u, T_v and $N(u, v)$ for the parametrized surfaces at the given points.

(1) $G(u, v) = (u^2 - v^2, u + v, u - v)$ at $(u, v) = (2, 3)$.

(2) $G(r, \theta) = (r\cos\theta, r\sin\theta, 1 - r^2)$ at $(r, \theta) = (1/2, \pi/4)$.

(3) $G(u, v) = (u\cos v, u\sin v, u)$ at $(u, v) = (1/4, \pi/2)$.

(4) $G(u, v) = (u, v^3, u + v)$ at $(u, v) = (1, 1)$.

Exercise 4.

Compute the integrals $\displaystyle\int\int_{S_i} f_i(x, y, z)\,\mathrm{d}S$ where $S_i = G(D_i)$ are the surfaces given by the parametrizations from previous exercise, point (i), $i \in \{1, \dots, 4\}$ and the functions and D_i are the followings:

(1) $f_1(x, y, z) = y$ and $D_1 = [1, 2] \times [0, 1]$.

(2) $f_2(x, y, z) = x^2 + y^2 + z^2$ and $D_2 = [1, 2] \times [0, \pi/2]$.

(2) $f_3(x, y, z) = (x^2 - y^2)z$ and $D_3 = [0, 2] \times [0, \pi/4]$.

(2) $f_4(x, y, z) = y$ and $D_4 = [1, 2] \times [1, 2]$.