Math 32B-1: Calculus of Several Variables

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First Midterm, January 27, 2016

Name (use a pen):

Student ID (use a pen):

Signature (use a pen):

Discussion section (choose one): 1A 1B 1C 1D

Rules:

- Duration of the exam: **50 minutes**.
- By writing your name and signature on this exam paper, you attest that you are the person indicated and will adhere to the UCLA Student Conduct Code.
- **No** calculators, computers, cell phones (all the cell phones should be turned off during the exam), notes, books or other outside material are permitted on this exam. If you want to use scratch paper, you should ask for it from one of the supervisors. Do not use your own scratch paper!
- Please justify all your answers with mathematical precision. You may loose points in the lack of justification of your answers.
- Theorems from the lectures may be used in order to justify your solution. In this case state the theorem you are using.
- This exam has **3 problems** and is worth **20 points**.
- I wish you success!

Problem	Score
Exercise 1	
Exercise 2	
Exercise 3	
Total	

Exercise 1 (7 points)**.**

Let D be the following two dimensional domain: $D := \{(x,y) \in \mathbb{R}^2 : \; 1 \leq x \leq e; 0 \leq y \leq \ln(x)\}$, where ln denotes the natural logarithm and the number e denotes its base, i.e. $e \approx 2.71$.

(1) Sketch the domain D and compute its area.

(2) Let us define the domain $\widetilde{D}$ as the part of the rectangle $[1,e] \times [0,1]$ that is above D. Sketch $\widetilde{D}$ and compute its area.

(3) Compute $\displaystyle\int\int_D \frac{e^{2y}}{x} \, \mathrm{d}A(x,y)$.

Now rotate the domain D around the $x-$axes to obtain a solid 3D object which is axially symmetric w.r.t. the $x-$axes. Call this domain W.

(4) Sketch the domain W and compute its volume.

Hint: find out what are the cross sections of this object if you cut it with planes parallel to the $y-$axis.

(5) Compute $\displaystyle\int\int\int_W yz \, \mathrm{d}V(x,y,z)$.

Hint: use the previously determined cross sections and use eventually polar coordinates in the $yz-$plane.

Exercise 2 (6 points)**.**

(1) Sketch the domains of integration and compute the following difference

$$\int_0^2 \int_{x/2}^1 \cos(y^2)\,\mathrm{d}y\,\mathrm{d}x - \int_0^1 \int_x^1 \cos(y^2)\,\mathrm{d}y\,\mathrm{d}x.$$

Hint: interchange the order of integration in both integrals. You may write the difference after as a single integral.

(2) Sketch the domain of integration, find an appropriate change of variable and compute

$$\int\int_D \frac{1}{\cos^2(x^2+y^2)}\,\mathrm{d}A(x,y),$$

where $D = \left\{\pi/6 \le x^2 + y^2 \le \pi/4\right\}$.

Exercise 3 (7 points).

Let us consider the following disks $C_1 := \{(x,y) \in \mathbb{R}^2 : \ x^2+y^2 \leq 1\}$ and $C_2 := \{(x,y) \in \mathbb{R}^2 : x^2+(y-\sqrt{2})^2 \leq 1\}$. We consider moreover the following domains:

- D_1 is bounded by the lines $y = x$, $y = \sqrt{2} - x$, $y = \sqrt{2} + x$ and $y = -x$;
- $D_2 = D_1 \cap C_1$, a circular sector of the disk C_1;
- $D_3 = D_1 \cap C_2$, a circular sector of the disk C_2;
- $D_4 = D_1 \setminus D_2$, the part of D_1 above D_2.
- $D_5 = C_1 \cap C_2$, the intersection of the disks C_1 and C_2;

Let us consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined as $f(x,y) = x^2 + y^2$.

(1) Sketch the domain of D_1 and the disks C_1 and C_2, then compute $\displaystyle\int\int_{D_1} f(x,y)\,\mathrm{d}A(x,y)$.

Hint: you may use the change of variables formula with the mapping $G(u,v) = \left(\frac{\sqrt{2}}{2}u + \frac{\sqrt{2}}{2}v, -\frac{\sqrt{2}}{2}u + \frac{\sqrt{2}}{2}v\right)$.
In this case show that D_1 is the image of the square $\widetilde{D}_1 = [-1,0] \times [0,1]$ through G.

(2) Sketch the domain D_2. Using eventually polar coordinates, compute $\displaystyle\int\int_{D_2} f(x,y)\,\mathrm{d}A(x,y)$.

(3) Sketch the domain D_3. Using eventually polar coordinates, compute $\displaystyle\int\int_{D_3} f(x,y)\,\mathrm{d}A(x,y)$.

(4) Sketch D_4 and compute $\displaystyle\int\int_{D_4} f(x,y)\,\mathrm{d}A(x,y)$.

(5) Sketch D_5 and compute $\displaystyle\int\int_{D_5} f(x,y)\,\mathrm{d}A(x,y)$.

Hint: in the last two points you may use the additive property of the integral with respect to decompositions of a domain.