Math 131B-1: Analysis – Homework 4

Due: May 3, 2017

Exercise 1 (Based on Tao)**.**

(1) Let $(\mathbb{R}, d)$ be the real line with the standard metric. Give an example of a continuous function $f : \mathbb{R} \to \mathbb{R}$ and an open set $V \subseteq \mathbb{R}$ such that the image set $f(V)$ is not open.

(2) The same situation as before, now give an example of a continuous function f and a closed set $F \subseteq \mathbb{R}$ such that the image set $f(F)$ is not closed.

(3) Let (X, d_X) and (Y, d_Y) be two metric spaces. Let $f : X \to Y$ be a function. Prove that the following statements are equivalent:

   (a) f is continuous.

   (b) Whenever F is a closed set in Y, the set $f^{-1}(F)$ is a closed set in X.

Exercise 2.

(1) Exercise 1.5.8 from Tao.

(2) Exercise 1.5.9 from Tao. A limit point of a sequence is the limit of a convergent subsequence of the original sequence.

(3) Exercise 1.5.10(c) from Tao.

In the following exercises you should use the standard Euclidean metric on $\mathbb{R}^2$.

Exercise 3 (From Tao)**.**

(1) Exercise 2.2.9. from Tao. (Do not discuss the comparison with Exercise 1.2.7!)

(2) Exercise 2.3.5. from Tao.

Exercise 4 (From Tao)**.**

(1) Exercise 2.2.10. from Tao.

(2) Exercise 2.2.11. from Tao.