Math 131B-1: Analysis – Homework 8

Due: May 31, 2017

Exercise 1.

(1) Exercise 4.1.2 for Tao (page 78 in my version).

(2) Exercise 4.2.1 from Tao (page 82 in my version).

(3) Exercise 4.2.6 from Tao (page 82 in my version).

Exercise 2.

(1) Exercise 4.2.2 from Tao (page 82 in my version).

(2) Show Lemma 4.3.2 (Exercise 4.3.1) from Tao (pages 84 and 86 in my version).

(3) Discuss about the relation between Lemma 4.3.2 and the integration by parts formula for real valued real functions.

(4) Discuss how the result of Abel's theorem (where we proved the continuity of the power series at the boundary point $a+R$) can be easily applied for the boundary point $a-R$.

Exercise 3.

(1) Find the radius of convergence for the following power series

$$(a)\ \sum_{n\geq 1} \frac{5^{2n}}{n^2}(x-3)^n;\quad (b) \sum_{n\geq 0} \frac{1}{n!}(x-1)^n;\quad (c) \sum_{n\geq 0} \sin(n\pi)(x-10)^n;\quad (d)\ \sum_{n=2}^{\infty} \frac{n+4}{n^2-1}(x-3)^{n+1}.$$

(2) Construct power series with the following intervals of converge, or explain why one does not exists.

$$(a)\ [0,1];\quad (b)\ (-\infty,+\infty);\quad (c)\ (-2,2);\quad (d)\ [-2,2).$$

Exercise 4.

(1) Suppose that the radius of convergence of $\sum_{n=0}^{+\infty} c_n x^n$ is $R=6$. What is the radius of convergence of $\sum_{n=3}^{+\infty} \frac{c_n}{3^{2n}} x^n$?

(2) Suppose that the radius of convergence of $\sum_{n=0}^{+\infty} c_n x^n$ is $R<1$. What can we say about the radius of convergence of the power series $\sum_{n=0}^{+\infty} s_n x^n$, where $s_n = c_0 + \cdots + c_n$?

(3) Prove that if the power series $\sum_{n\geq 1} c_n (x-a)^n$ diverges for $x=b$, then it diverges for all x whenever $|x-a|>|b-a|$.

Exercise 5.

Exercise 4.2.8 from Tao (page 83).