Math 32B-2: Calculus of Several Variables – Homework #5

Instructor: Alpár R. Mészáros

Due: May 3, 2019

Exercise 1 (From Rogawski-Adams)**.**

(1) Use spherical coordinates to find the volume of a sphere of radius 2 from which a central cylinder of radius 1 has been removed.

For (2) and (3) use spherical coordinates to calculate the triple integral of $f(x, y, z)$ over the given region.

(2) $f(x, y, z) = (x^2 + y^2 + z^2)^{-3/2}$; $2 \le x^2 + y^2 + z^2 \le 4$

(3) $f(x, y, z) = 1$; $x^2 + y^2 + z^2 \le 4$, $z \ge \sqrt{x^2 + y^2}$

Exercise 2 (From Rogawski-Adams)**.**

Use cylindrical coordinates to calculate the triple integral for the given function and region

(1) $f(x, y, z) = z\sqrt{x^2 + y^2}$; $x^2 + y^2 \le z \le 8 - (x^2 + y^2)$.

(2) $f(x, y, z) = z$; $0 \le z \le x^2 + y^2 \le 9$

Express the triple integral in cylindrical coordinates

(3) $\displaystyle\int_0^1 \int_{y=-\sqrt{1-x^2}}^{y=\sqrt{1-x^2}} \int_{z=0}^{4} f(x, y, z)\,\mathrm{d}z\,\mathrm{d}y\,\mathrm{d}x$

(4) $\displaystyle\int_0^2 \int_{y=0}^{y=\sqrt{2x-x^2}} \int_{z=0}^{\sqrt{x^2+y^2}} f(x, y, z)\,\mathrm{d}z\,\mathrm{d}y\,\mathrm{d}x$

Exercise 3 (From Rogawski-Adams)**.**

(1) Sketch an example of a non-constant vector field in the plane in which each vector is parallel to the vector $(1; 1)$.

(2) Give an example of a potential function for the vector field $F(x, y, z) = (yz, xz, xy)$ other than $f(x, y, z) = xyz$.

(3) Sketch the following vector fields in the plane by drawing the vectors attached to points with integer coordinates in the rectangle $-4 \le x \le 4$ and $-3 \le y \le 3$. $F_1(x, y) = (1/2; 1/2)$; $F_2(x, y) = \left(-\frac{y}{\sqrt{x^2 + y^2}}; -\frac{x}{\sqrt{x^2 + y^2}}\right)$; $F_3(x, y) = (2x + 2; y)$.

(4) Compute the divergence and the curl of the following vector fields: $F_1(x, y) = (y/x; y/z; z/x)$; $F_2(x, y, z) = (e^{xy}; \cos(xy); \sin(z))$; $F_3(x, y, z) = (z^2; x^2; y^2)$.

Exercise 4 (From Rogawski-Adams)**.**

(1) The velocity vectors in km/h for the wind speed of a tornado near the ground are given by the vector field $F(x, y) = \left(-\frac{y}{e^{(x^2+y^2-1)^2}}, \frac{x}{e^{(x^2+y^2-1)^2}}\right)$. Determine the coordinates of those points where the wind speed is the highest.

(2) Find a potential function for the vector field $F(x, y) = (x, 0)$; and prove that $G(x, y) = (y, 0)$ is not conservative.

(3) Prove that $\mathrm{curl}(fF) = f\mathrm{curl}(F) + (\nabla f) \times F$, where f is a scalar function and F is a vector field.

(4) Show that $\mathrm{div}(fF) = \nabla f \cdot F + f\mathrm{div}(F)$, where f is a scalar function and F is a vector field.