Math 32B-1: Calculus of Several Variables – Homework #6

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Exercise 1 (Based on Rogawski-Adams).

Compute the line integrals $\int_C f\,\mathrm{d}s$ of the following functions on the specified curves

(1) $f(x,y) = \frac{y^3}{x^7}$; and C is given by $y = (1/4)x^4$ for $1 \le x \le 2$.

(2) $f(x,y,z) = 3x - 2y + z$; $r(t) = (2+t, 2-t, 2t)$ where $-2 \le t \le 1$.

(3) $f(x,y,z) = x^2 z$; $r(t) = (e^t, \sqrt{2}t, e^{-t})$ where $0 \le t \le 1$.

Exercise 2 (Based on Rogawski-Adams).

Compute the line integrals $\int_C F \cdot \mathrm{d}r$ of the following vector fields on the specified curves

(1) $F(x,y,z) = (z^2, x, y)$ and C is given by $r(t) = (3+5t^2; 3-t^2, t)$ for $0 \le t \le 2$.

(2) $F(x,y,z) = (xy, 2, z^3)$ on the Helix given by $r(t) = (\cos(t), \sin(t), t)$ for $t \in [0, \pi]$.

(3) $F(x,y) = (4, y)$ on the quarter of the circle $x^2 + y^2 = 1$ with $x \le 0, y \ge 0$, oriented clockwise.

Exercise 3 (Based on Rogawski-Adams).

Compute $\int_C F \cdot \mathrm{d}r$ for the following oriented curves:

(1) $F(x,y) = (e^{y-x}, e^{2x})$, piecewise linear path from $(1,1)$ to $(2,2)$ to $(0,2)$.

(2) $F(x,y) = \left(\frac{-y}{(x^2+y^2)^2}, \frac{x}{(x^2+y^2)^2}\right)$, circle of radius R with center at the origin oriented counter-clockwise.

(3) $F(x,y,z) = (z^3, yz, x)$, quarter of the circle of radius 2 in the yz-plane with center at the origin where $y \ge 0$ and $z \ge 0$, oriented clockwise when viewed from the positive x-axis.

(3) $F(x,y,z) = (e^z, e^{x-y}, e^y)$, the closed path between $(0,0,6)$, $(0,4,0)$ and $(2,0,0)$.

Exercise 4 (Based on Rogawski-Adams).

(1) Calculate the total mass of a metal tube in the helical shape $r(t) = (\cos t, \sin t, t^2)$ (distance in centimeters) for $0 \le t \le 2\pi$ if the mass density is $\delta(x,y,z) = \sqrt{z}$ g/cm.

(2) Calculate the work done by the field $F(x,y,z) = (x,y,z)$ when the object moves along $r = (\cos t, \sin t, t)$ for $0 \le t \le 3\pi$.

(3) Calculate the work done by the field $F(x,y,z) = (e^x, e^y, xyz)$ when the object moves along $r = (t^2, t, t/2)$ for $0 \le t \le 1$.

(4) Let $F(x,y) = (y,x)$. Prove that if C is any path from (a,b) to (c,d), then $\int_C F \cdot \mathrm{d}r = cd - ab$.