Math 32B-2: Calculus of Several Variables – Homework #7

Due: May 17, 2019

Exercise 1 (Based on Rogawski-Adams).

(1) Let $F(x, y, z) = (x^{-1}z, y^{-1}z, \ln(xy))$.

(a) Verify that $F = \nabla f$, where $f(x, y, z) = z \ln(xy)$.

(b) Evaluate $\int_C F \cdot \mathrm{d}r$, where $r(t) = (e^t, e^{2t}, t^2)$ for $1 \le t \le 3$.

(c) Evaluate $\int_C F \cdot \mathrm{d}r$, for any path from $P = (1/2, 4, 2)$ to $Q = (2, 2, 3)$ contained in the region $x > 0$, $y > 0$.

(d) Why is it necessary to specify that the path lies in the region where x and y are positive?

(2) Verify that $F = \nabla f$, where $F(x, y) = (\cos y, -x \sin y)$, $f(x, y) = x \cos y$ and evaluate the line integral of F over the upper half of the unit circle centered at the origin, oriented counterclockwise.

Exercise 2 (Based on Rogawski-Adams).

Find a potential function for F or determine that F is not conservative.

(1) $F_1(x, y, z) = (y, x, z^3)$, $F_2(x, y, z) = (\cos z, 2y, -x \sin z)$, $F_3(x, y, z) = (e^x(z+1), -\cos y, e^x)$, $F_4(x, y, z) = (yze^{xy}, xze^{xy} - z, e^{xy} - y)$.

(2) Let $F(x, y) = \left(\frac{1}{x}, \frac{-1}{y}\right)$. Calculate the work against F required to move from $(1, 1)$ to $(3, 4)$ along any path in the first quadrant.

Exercise 3 (Based on Rogawski-Adams).

(1) What is the outward-pointing unit normal to the sphere of radius 3 centered at $P = (2, 2, 1)$?.

(2) Show that $G(r, \theta) = (r \cos \theta, r \sin \theta, 1 - r^2))$ parametrizes the paraboloid $z = 1 - x^2 - y^2$. Describe the grid curves of this parametrization.

(3) Let $S = G(D)$, where $D = \{(u, v) : u^2 + v^2 \le 1; u \ge 0, v \ge 0\}$ and $G(u, v) = (2u + 1, u - v, 3u + v)$. Compute the surface area of S.

Exercise 4 (Based on Rogawski-Adams).

Compute T_u, T_v and $N(u, v)$ for the parametrized surfaces at the given points.

(1) $G(u, v) = (u^2 - v^2, u + v, u - v)$ at $(u, v) = (2, 3)$.

(2) $G(r, \theta) = (r \cos \theta, r \sin \theta, 1 - r^2)$ at $(r, \theta) = (1/2, \pi/4)$.

(3) $G(u, v) = (u \cos v, u \sin v, u)$ at $(u, v) = (1/4, \pi/2)$.

(4) $G(u, v) = (u, v^3, u + v)$ at $(u, v) = (1, 1)$.