1 Let $f(z) = \left(-1 - i\sqrt{3}\right) z + 3 - 2i$. Describe $f$ as a rotation followed by a dilation followed by a translation. Hence draw the image under $f$ of the unit circle $|z| = 1$ and of the line $x = y$.

2 Show that the function $f(z) = az + b$ (with $a \neq 0$) may be described as a translation followed by a rotation followed by a dilation. Describe $f(z) = \sqrt{2}\,(1 - i)\, z + 2 - 4i$ in this way.

3 In what subset of the complex plane is $4z^3 - 3iz^2 + 4 - 3i$ conformal?

4 In what subset of the complex plane is $2z^3 - 3\,(1 + i)\, z^2 + 6iz$ conformal?

5 In what subset of the complex plane is $\sinh z$ conformal? For every point $z_0$ at which the function is not conformal, give an example of two paths (lines) through $z_0$ such that the angle (or the orientation of the angle) between them is not preserved by $f(z)$ at $z_0$.

6 At which points in $\mathbb{C}$ are the following maps conformal?
(a) $f(z) = z^3 + 2i$ (b) $f(x + iy) = x - 3yi$
In both cases, for every point $z_0$ at which the function is not conformal, give an example of two paths (lines) through $z_0$ such that the angle (or the orientation of the angle) between them is not preserved by $f(z)$ at $z_0$.

7 Let $f(z) = z^2 + 2z$. Show that $f$ is conformal at $z = i$ and describe the effect of $f'(z)$ on the tangent vectors of curves passing through this point.

8 Let $f(z) = 2z^3 + 3z^2$. Show that $f$ is conformal at $z = i$ and describe the effect of $f'(z)$ on the tangent vectors of curves passing through this point.

9 Is the following true or false? If $f, g$ are conformal at a point $z_0$ then $f + g$ is conformal at $z_0$. Give a proof or a counter-example.

10 Let $f(z) = \overline{g(z)}$ with $g(z)$ holomorphic (such functions $f$ we call **anti-holomorphic**). Describe geometrically what happens to tangent vectors of paths passing through a point under the map $f$. Conclude that $f$ is angle-preserving, but reverses the orientation.

11 Let $f : \mathcal{D} \to \mathcal{D}'$ be a biholomorphic map between two domains $\mathcal{D}$ and $\mathcal{D}'$. By considering the equation $f(f^{-1}(w)) = w$ (for $w \in \mathcal{D}'$), show that $f$ is conformal.

12 Let $\Omega := \{z \in \mathbb{C} : \mathrm{Re}(z) > \sqrt{3}\,|\,\mathrm{Im}(z)|\}$. Sketch the domain $\Omega$ and find its image $f(\Omega)$ under the map $f(z) = z^6$. Hence show that $f$ is a biholomorphic map from $\Omega$ onto its image, and give the inverse function.