1 Is there a non-trivial Möbius transformation from the upper half-plane to itself that fixes both the points $1+i$ and $-1+i$? Either find an example of such a map or show that none exists.

2 Find an automorphism of the unit disc that takes $\frac{1}{2}$ to 0 and (when considered as a map $\hat{\mathbb{C}} \to \hat{\mathbb{C}}$) also takes -1 to $-i$.

3 Find an automorphism of the unit disc that takes $-\frac{i}{2}$ to 0 and (when considered as a map $\hat{\mathbb{C}} \to \hat{\mathbb{C}}$) also takes 1 to i.

4 Find a Möbius transformation f from the upper half-plane $\mathbb{H}$ onto the unit disc $\mathbb{D}$ that takes $1+i$ to 0 and (when considered as a map $\hat{\mathbb{C}} \to \hat{\mathbb{C}}$) also takes 1 to $-i$. Give an explicit formula for $f(z)$.

5 Find a Möbius transformation f from the unit disc $\mathbb{D}$ onto the upper half-plane $\mathbb{H}$ that takes 0 to i and (when considered as a map $\hat{\mathbb{C}} \to \hat{\mathbb{C}}$) also takes i to 2. Give an explicit formula for $f(z)$.

6 Let $\Omega \subset \mathbb{C}$ be the set of all complex numbers z for which $\mathrm{Re}(z) > -1$ and $\mathrm{Im}(z) > -1$.
(a) Find a biholomorphic map f from Ω onto the upper half-plane. Give an explicit formula for $f(z)$.
(b) Hence, find a biholomorphic map $\tilde{f}$ from Ω onto the open unit disc. Give an explicit formula for $\tilde{f}(z)$.

7 Does there exist a biholomorphic map taking the closed upper half of the unit disc onto the closed unit disc; i.e. from $\{z \in \mathbb{C} : |z| \leq 1,\ \mathrm{Im}\, z \geq 0\}$ onto $\{w \in \mathbb{C} : |w| \leq 1\}$? Either find an example of such a map or show that none exists. *Hint:* The boundary would be mapped to the boundary.

8 Use standard examples to find a biholomorphic map from the upper half $\Omega := \{z \in \mathbb{D} : \mathrm{Im}(z) > 0\}$ of the unit disc onto the unit disc $\mathbb{D}$.

9 Consider the map $z \to z^2$.
(i) Find and sketch the images of the lines $\mathrm{Im}\, z = b$ (for $0 < b < 1$).
Hint: find a parametrisation for the lines.
(ii) Find the image of $\{z : 0 < \mathrm{Im}\, z < 1\}$ under this map.

10 Describe the image of
(i) $\{z : |z-1| > 1\}$ under $z \to w = \frac{z}{z-2}$
(ii) $\{z : |z-i| < 1, \mathrm{Re}\, z < 0\}$ under $z \to w = \frac{z-2i}{z}$

11 Construct a biholomorphic map f from $\mathcal{R}$ onto $\mathcal{R}'$, where $\mathcal{R} = \{z : \mathrm{Im}\, z < \frac{1}{2}\}$ and $\mathcal{R}' = \{z : |z-1| < 2\}$. Give an explicit formula for $f(z)$.

12 (a) Find the unique Möbius transformation $f(z)$ taking the ordered set of points $\{0, -1, -i\}$ to the ordered set of points $\{1, \infty, i\}$ in $\hat{\mathbb{C}}$.
(b) Let C_1 be the circle through 0, -1 and i, and let C_2 be the circle through 0, -1 and $-i$. Let $\mathcal{R}$ be the intersection of the interiors of the two circles. Find the image of $\mathcal{R}$ under your map f, and hence construct a biholomorphic map from $\mathcal{R}$ to the set $\Omega := \{w \in \mathbb{C} : -\pi/4 < \mathrm{Arg}(w) < \pi/4\}$.
(c) Find a biholomorphic map from $\mathcal{R}$ to the upper half-plane $\mathbb{H}$.

13 Consider the region $P \subset \mathbb{C}$ defined by $P = \left\{ z \in \mathbb{D} \ : \ -3\pi/4 < \mathrm{Arg}\, z < 3\pi/4 \right\}$.
(a) Draw P in the complex plane.
(b) Find a biholomorphic map from P onto the upper half-plane $\mathbb{H}$.
(c) Find a biholomorphic map from P onto the lower half-plane $\mathbb{H}_L := \{w \in \mathbb{C} : \mathrm{Im}(z) < 0\}$.