1 Find parametrisations for the following curves:
(a) The line segment from -1 to $1-i$,
(b) The circle of radius 2 centred at $1+i$,
(c) The ellipse $x^2+4y^2=1$.

2 Let $\gamma:[a,b]\to\mathbb{C}$ be a C^1-curve, and define $(-\gamma):[-b,-a]\to\mathbb{C}$, by $(-\gamma)(t):=\gamma(-t)$. Show that for any f such that $\int_\gamma f(z)\,dz$ is well defined we have that

$$\int_{-\gamma} f(z)\,dz = -\int_\gamma f(z)\,dz$$

3 Let $\gamma:[0,4]\to\mathbb{C}$ be the curve given by

$$\gamma(t)=\begin{cases} t, & 0\le t\le 1,\\ 1+i(t-1), & 1\le t\le 2,\\ 3-t+i, & 2\le t\le 3,\\ i(4-t), & 3\le t\le 4.\end{cases}$$

Draw this curve in the complex plane and directly compute $\int_\gamma e^z dz$ (without using the Fundamental Theorem of Calculus).

4 Calculate $\int_\gamma |z|\,dz$ when γ is the straight line from $-i$ to i, and when γ is the segment of the unit circle which joins $-i$ to i in the right hand half-plane.

5 Calculate $\int_\gamma \frac{1}{z}\,dz$, where $\gamma(t)=(1+2t)e^{4\pi i t}$ for $0\le t\le 1$.

6 Let γ_ρ be the curve $\gamma_\rho(\theta):=\rho e^{i\theta}$ with $0\le\theta\le\pi$. Let $z^{\frac{1}{2}}$ be the branch of square root corresponding to the branch of $\log$ with argument in $(-\pi/2,3\pi/2]$, that is, if $z=\rho e^{i\theta}$ with $\theta\in(-\pi/2,3\pi/2]$ then $z^{\frac{1}{2}}=\sqrt{\rho}e^{i\theta/2}$. Show that

$$\lim_{\rho\to\infty}\int_{\gamma_\rho}\frac{z^{1/2}}{z^2+1}=0.$$

7 Let γ be any piecewise C^1-curve from -3 to 3 such that, except for the end points, lies entirely in the upper half plane. Calculate the integral

$$\int_\gamma f(z)\,dz,$$

where $f(z)$ is the branch of $z^{\frac{1}{2}}$ defined by $\sqrt{r}e^{i\theta/2}$ with $0<\theta<2\pi$.