Q.1 *Let X be a set and let $\mathrm{d} : X \times X \to \mathbb{R}$ be defined as*

$$\mathrm{d}(x, y) = \begin{cases} 0, & x = y, \\ 1, & x \neq y. \end{cases}$$

Show that d is a metric on X (which we call the discrete metric).
Show in addition that if X is a non-trivial vector space, the discrete metric can't be induced by a norm. In other words, show that there exists no norm on X, $\|\cdot\|$, such that

$$\mathrm{d}(x, y) = \|x - y\| .$$

Q.2 *(Assignment sheet 2 problem 3) In the space* $C([a,b])$ *of continuous functions defined on a closed interval* $[a,b]$ *(for* $a<b$*), let*

$$d_1(f,g) := \int_a^b |f(t) - g(t)|\, dt.$$

Show that d_1 *is a metric on* $C([a,b])$.

Q.3 *(Assignment sheet 2 problem 7)*

(i) *Show that in any metric space* (X, d) *the set* $\{x\}$*, consisting of a single point* $x \in X$*, is closed.*

(ii) *Show that in any metric space* (X, d) *the closed ball* $\overline{B}_r(x) := \{y \in X : d(y, x) \leq r\}$*, of radius* $r > 0$ *centred at* $x \in X$*, is closed.*

Q.4 *(Assignment sheet 2 problem 13) Give an example of a metric space X and an $x \in X$ such that $\overline{B}_1(x) \neq \overline{B_1(x)}$; that is, the closure of the open ball is not necessarily the closed ball!!*