

Q.1 Show the following statement from class: Let (X, d) be a metric space and let $A \subseteq X$. Then A is closed if and only if any sequence of elements of A that converges has its limit in A . In other words if $x_n \in A$ for all $n \in \mathbb{N}$ and $\lim_{n \rightarrow \infty} x_n = x$ then $x \in A$.

Q.2 (i) (Assignment sheet 2 problem 15) Show that if a sequence $\{x_n\}$ converges in a discrete metric space, then it is eventually constant.

(ii) Find a metric space (X, d) and a set $A \subseteq X$ such that A is closed and bounded but is not sequentially compact.

Q.3 (Assignment sheet 4 problem 4) The real axis and the imaginary axis divide \mathbb{C} into four quadrants as follows:

$$\begin{array}{c|c} \Omega_2 & \Omega_1 \\ \hline \Omega_3 & \Omega_4 \end{array}$$

Determine the images of $\Omega_1, \Omega_2, \Omega_3, \Omega_4$ under the exponential map $z \mapsto e^z$.

Q.4 (Assignment sheet 4 problem 6(b)) Give examples to illustrate that, in general, for complex numbers z, w ,

$$\text{Log}(zw) \neq \text{Log}(z) + \text{Log}(w).$$