Q.1 *(Assignment sheet 4 problem 17) Let $f(z)$ be a holomorphic function. Prove the following variants of the* **Zero derivative theorem***, which says that, if any one of the following conditions hold on a domain D then $f(z)$ is constant on D.*
(i) $f(z)$ is a real number for all $z \in D$.
(ii) the real part of $f(z)$ is constant on D.
(iii) the modulus of $f(z)$ is constant on D.

Q.2 *(Assignment sheet 5 problem 5) In what subset of the complex plane is* $\sinh z$ *conformal? For every point* z_0 *at which the function is not conformal, give an example of two paths (lines) through* z_0 *such that the angle (or the orientation of the angle) between them is not preserved by* $f(z)$ *at* z_0.

Q.3 *(Assignment sheet 6 problem 5) Is there a Möbius transformation which maps the sides of the triangle with vertices at* $-1, i$ *and* 1 *to the sides of an equilateral triangle (all sides of equal length)? Either give an example of such a Möbius transformation, or explain why it is not possible.*

Q.4 *(Assignment sheet 6 problem 8) If α and β are the two fixed points of a Möbius transformation $f(z)$, show that for all $z \neq \alpha, \beta$ and $f(z) \neq \infty$, we have*

$$\frac{f(z)-\alpha}{f(z)-\beta} = K\frac{z-\alpha}{z-\beta},$$

where K is a constant.

Q.5 *(Assignment sheet 6 problem 9) Find the Möbius transformation taking the ordered set of points* $\{-i, -1, i\}$ *to the ordered set of points* $\{-i, 0, i\}$. *What is the image of the unit disc under this map? Which point is sent to* ∞*?*