Q.1 *(Assignment sheet 7 problem 4) Find a Möbius transformation* $f$ *from the upper half-plane* $\mathbb{H}$ *onto the unit disc* $\mathbb{D}$ *that takes* $1+i$ *to* $0$ *and (when considered as a map* $\hat{\mathbb{C}} \to \hat{\mathbb{C}}$*) also takes* $1$ *to* $-i$*. Give an explicit formula for* $f(z)$*.*

Q.2 *(Assignment sheet 7 problem 8) Use standard examples to find a biholomorphic map from the upper half* $\Omega := \{z \in \mathbb{D} : \mathit{Im}(z) > 0\}$ *of the unit disc onto the unit disc* $\mathbb{D}$.

Q.3 *(Assignment sheet 7 problem 11) Construct a biholomorphic map $f$ from $\mathcal{R}$ onto $\mathcal{R}'$, where $\mathcal{R} = \{z : \mathit{Im}\, z < \frac{1}{2}\}$ and $\mathcal{R}' = \{z \, : \, |z - 1| < 2\}$. Give an explicit formula for $f(z)$.*

Q.4 *(Assignment sheet 8 problem 3)*

*(i) Show that for any* $\rho > 0$ *the sequence* $\left\{\frac{1}{nz}\right\}_{n\in\mathbb{N}}$ *converges uniformly on* $A = \{z \in \mathbb{C} : \ |z| \geq \rho\}$.

*(ii) Does* $\left\{\frac{1}{nz}\right\}_{n\in\mathbb{N}}$ *converge uniformly on* $\mathbb{C}^* := \mathbb{C} \setminus \{0\}$*?*

Q.5 *(Assignment sheet 8 problem 6) For every $n \in \mathbb{N}$, let $f_n(z) = \sin(z/n)$ for $z \in \mathbb{C}$. Show that $\{f_n\}_{n\in\mathbb{N}}$ converges pointwise on $\mathbb{C}$. Let $\rho$ be a positive real number. Show that $\{f_n\}_{n\in\mathbb{N}}$ converges uniformly on $A = \{z : |z| \leq \rho\}$. Show that $\{f_n\}_{n\in\mathbb{N}}$ does not converge uniformly on $\mathbb{C}$.*

Q.6 *(Assignment sheet 8 problem 9) Prove that* $\sum_{n=0}^{\infty} e^{nz}$ *converges uniformly on* $A = \{z \in \mathbb{C} : \textit{Re}(z) \leq -1\}$, *but not on* $B = \{z \in \mathbb{C} : \textit{Re}(z) \leq 0\}$.

Q.7 *(Assignment sheet 8 problem 10) Let* $R$ *satisfy* $0 < R < 1$. *Show that the series* $\sum_{n=1}^{\infty} \frac{z^n}{1+z^n}$ *converges uniformly on* $A = \{z \in \mathbb{C} : |z| < R\}$. *Conclude that the infinite series defines a continuous function on the unit disc* $\mathbb{D}$.