Q.1 *Show that for any* $a \in \mathbb{C}$ *the curve* $\gamma(t) = e^{\alpha t}$ *satisfies*

$$\gamma'(t) = \alpha e^{\alpha t} (= \alpha \gamma(t)).$$

Conclude that for any $-\infty < a < b < \infty$ *and any* $\alpha \neq 0$

$$\int_a^b e^{\alpha t} dt = \frac{e^{\alpha b} - e^{\alpha a}}{\alpha}.$$

Q.2 *(Assignment sheet 9 problem 5) Calculate* $\int_\gamma \frac{1}{z}\,dz$*, where* $\gamma(t) = (1+2t)e^{4\pi i t}$ *for* $0 \le t \le 1$.

Q.3 *(Assignment sheet 9 problem 6) Let* γ_ρ *be the curve* $\gamma_\rho(\theta) := \rho e^{i\theta}$ *with* $0 \leq \theta \leq \pi$. *Let* $z^{\frac{1}{2}}$ *be the branch of square root corresponding to the branch of* $\log$ *with argument in* $(-\pi/2, 3\pi/2)$, *that is, if* $z = \rho e^{i\theta}$ *with* $\theta \in (-\pi/2, 3\pi/2)$ *then* $z^{\frac{1}{2}} = \sqrt{\rho} e^{i\theta/2}$. *Show that*

$$\lim_{\rho \to \infty} \int_{\gamma_\rho} \frac{z^{1/2}}{z^2+1} dz = 0.$$

Q.4 *(Assignment sheet 9 problem 7) Let γ be any piecewise C^1-curve from -3 to 3 such that, except for the end points, lies entirely in the upper half plane. Calculate the integral*

$$\int_\gamma f(z)\,dz,$$

where $f(z)$ is the branch of $z^{\frac{1}{2}}$ defined by $\sqrt{r}e^{i\theta/2}$ with $0 < \theta < 2\pi$.