

Problem Class 2

Exercise 1. Show that $\mathcal{B} = \{1, x, \dots\}$ is not a Schauder basis for $C[a, b]$.

Exercise 2. Show that if \mathcal{H} is an inner product space that has an uncountable orthonormal set then it can't be separable.

Exercise 3. Consider the Schauder basis $\mathcal{B} = \{\mathbf{e}_n\}_{n \in \mathbb{N}}$ for $\ell_p(\mathbb{N})$, $p \in [1, \infty)$, defined by

$$(\mathbf{e}_n)_k = \begin{cases} 1 & k = n \\ 0 & k \neq n \end{cases}.$$

Define the vectors

$$\mathbf{x}_n = \begin{cases} \mathbf{e}_1 & n = 1 \\ \mathbf{e}_n - \mathbf{e}_{n-1} & n \geq 2 \end{cases}.$$

Show that $\widetilde{\mathcal{B}} = \{\mathbf{x}_n\}_{n \in \mathbb{N}}$ is a Schauder basis for $\ell_1(\mathbb{N})$.