1) Consider a Lorentz vector with components x^μ, which transforms under Lorentz transformations as

$$x^\mu \to x'^\mu = \Lambda^\mu{}_\nu x^\nu \, .$$

Note that throughout this problem we are using summation convention.

a) Let $f^{\mu\nu} \equiv x^\mu x^\nu$. Find the transformation behavior of $f^{\mu\nu}$, $f^\mu{}_\nu = x^\mu x_\nu$ and $f_{\mu\nu} = x_\mu x_\nu$ under Lorentz transformations.

b) For another Lorentz vector y^μ, find the transformation behavior of $f^{\mu\nu} y_\mu$ under Lorentz transformations.

c) Compute

$$\sum_\mu \frac{\partial}{\partial x^\mu} x^\mu \, .$$

d) Work out the transformation behavior of

$$\frac{\partial}{\partial x^\mu}$$

under Lorentz transformations. Use c) to argue for the same result.

2) Write a 4-vector (x^0, x^1, x^2, x^3) as a matrix M_x with $M_x^\dagger = M_x$:

$$M_x := \begin{pmatrix} x^0 + x^3 & x^1 - ix^2 \\ x^1 + ix^2 & x^0 - x^3 \end{pmatrix} \, . \tag{0.1}$$

For $g \in SL(2,\mathbb{C})$ define an action $F(g)$ on $\mathbb{R}^4$ by

$$g \to F(g) \qquad F(g) M_x := g M_x g^\dagger \, . \tag{0.2}$$

a) Show that F is a homomorphism from $SL(2,\mathbb{C})$ to L.

b) For a rotation in the x^1, x^2-plane, find the element $g \in SL(2,\mathbb{C})$ that is mapped to it by F. Repeat the same for a boost along the x^1 direction.

Here are some things to ponder:

1. How is the Lorent group defined? Why is it defined that way?
2. What's the point about upper/lower indices?