3) Verify that

$$l^{01} = \begin{pmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad l^{02} = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad l^{03} = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$$
$$l^{12} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad l^{13} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix} \quad l^{23} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix} \tag{0.1}$$

are in the Lie algebra of L.

4) The Dirac matrices are

$$\gamma^0 = \begin{pmatrix} 0 & \mathbb{1}_{2\times 2} \\ -\mathbb{1}_{2\times 2} & 0 \end{pmatrix}, \qquad \gamma^i = \begin{pmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{pmatrix} \quad i = 1,2,3 \tag{0.2}$$

where $\mathbb{1}_{2\times 2}$ is the 2×2 identity matrix and σ_i are the Pauli matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{0.3}$$

a) Show that the Dirac matrices obey $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}\mathbb{1}_{4\times 4}$.

b) Show the 'freshers dream':

$$(a_\mu \gamma^\mu)^2 = a_\mu a^\mu \mathbb{1}_{4\times 4} \tag{0.4}$$

5) Using the Dirac matrices, show that $S^{\mu\nu} = \frac{1}{4}[\gamma^\mu, \gamma^\nu]$ are equal to

$$S^{0i} = \tfrac{1}{2}\begin{pmatrix} \sigma_i & 0 \\ 0 & -\sigma_i \end{pmatrix}, \qquad S^{jk} = \frac{i}{2}\epsilon_{jkl}\begin{pmatrix} \sigma_l & 0 \\ 0 & \sigma_l \end{pmatrix} \tag{0.5}$$

where i, j, k only take values $1, 2, 3$. What does this imply about the reducibility of the representation of $SL(2,\mathbb{C})$ defined by exponentiating the $S^{\mu\nu}$?

Here are some things to ponder:

1. What is the global structure of the Lorentz group?
2. How can we construct a representation of the Lie algebra of L using the Dirac matrices?