6) Verify that

   a) For an element $\Lambda(\theta) = e^{l^{12}\theta}$ of the Lorentz group (l^{12} is one of the generators of the Lorentz algebra introduced in the lectures) show that $g(0) = g(2\pi) = \mathbb{1}$. Now compare this behavior to the corresponding element of the representation acting on a Dirac spinor: $\Lambda_{1/2}(\theta) = e^{S^{12}\theta}$.

   b) Let $\gamma^5 := i\gamma^0\gamma^1\gamma^2\gamma^3$. What is $\frac{1}{2}\left(\gamma^5 \pm \mathbb{1}\right)\Psi$ for Ψ a Dirac spinor written in terms of Weyl spinors?

7) How does
$$B^{\mu\nu} \equiv \bar{\Psi}\gamma^\mu\gamma^\nu\Psi$$
transform under Lorentz transformations for Ψ a Dirac spinor?

8) For a Dirac spinor Ψ write
$$\bar{\Psi}\gamma^\mu\Psi \qquad \text{and} \qquad \bar{\Psi}\Psi$$
in terms of Weyl spinors.

9 For a relativistic point particle moving on path C through space-time, the only Lorentz invariant property of C is its length. Taking the action of a relativistic particle to be the length of C and parametrizing C as $x^\mu(s)$ we can write this as
$$S[x^\mu, \dot{x}^\mu] = -cm \int_C ds = -cm \int_C \sqrt{-\dot{x}^\mu \dot{x}_\mu} ds\,. \tag{0.1}$$
for a constant m and c the speed of light and $\dot{x}^\mu = \partial/\partial s\ x^\mu$. C is called the world-line of the particle.

   a) Show that this action is invariant under Lorentz transformations.

   b) Find the equations of motions and show that they are solved by straight lines in space-time.

   c) Set $s = t$ and expand the action for slow particles to recover the action of a non-relativistic point particle.

Here are some things to ponder:

1. In which ways is the relationship between $SO(3)$ and $SU(2)$ the same as the relationship between the Lorentz group and $SL(2, \mathbb{C})$.
2. What is a spinor?
3. What is an action?