10) Consider the following action of a real scalar field

$$S = \int d^4x \, \partial_\mu \phi \partial^\mu \phi + m^2 \phi^2 \, .$$

Show that the equations of motion are

$$(-\partial_\mu \partial^\mu + m^2)\phi = 0 \, .$$

11) Consider the action

$$S = \int d^4x \bar{\Psi} \left(\gamma^\mu \partial_\mu + m \right) \Psi \, .$$

for a Dirac spinor Ψ.

a) Show that S is Lorentz invariant.
b) Find the equations of motion.
c) Find the conserved charge associated to the $U(1)$ symmetry $\Psi \to e^{i\theta}\Psi$.
d) Show that

$$\left(\gamma^\mu \partial_\mu - m \right) \left(\gamma^\nu \partial_\nu + m \right) = \partial_\mu \partial^\mu - m^2$$

12 Consider a field Φ transforming in the adjoint representation of the Lie group $SU(n)$. Show that

$$S = \int d^4x \ \mathrm{tr} \left(\partial_\mu \Phi \partial^\mu \Phi \right)$$

is invariant under the action of $SU(n)$ and find the associated conserved current.

Here are some things to ponder:

1. What is an action?
2. What is a symmetry of an action?
3. When do we consider a physical system to be Lorentz invariant?