13) Show that using the field strength $F_{\mu\nu}$ and the 4-current J^μ we can write the Maxwell equations as

$$\partial_\nu F^{\mu\nu} = J^\mu \ , \qquad \epsilon^{\mu\nu\rho\sigma}\partial_\nu F_{\rho\sigma} = 0 \ .$$

14) Show that

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

can be written as

$$\boldsymbol{E} = -\nabla\phi - \frac{\partial \boldsymbol{A}}{\partial t} \ , \qquad \boldsymbol{B} = \nabla \times \boldsymbol{A} \ .$$

15) Show

$$\frac{\partial}{\partial X_{a_1\ldots a_n}}(X^{b_1\ldots b_n}X_{b_1\ldots b_n}) = 2X^{a_1a_2\ldots a_n} \ ,$$

for any tensor X with components $X_{a_1\ldots a_n}$.

Here are some things to ponder:

1. How do electric and magnetic fields behave under Lorentz transformations?
2. Which action reproduces the Maxwell equations?
3. What is the relationship of the potential A_μ to observable physics?