1) Let $\mathbb{C}$ be the complex numbers and $\mathbb{C}^* = \mathbb{C} \setminus \{0\}$. Which of these is a group under addition? Which of these is a group under multiplication?

2) Consider the set V of real $n \times n$ matrices.

   a) Show that V is a (real) vector space.

   b) Let $U \subset V$ be the set of matrices with determinant 1. Is U a vector space as well?

   c) For any matrix Q in V define a map

   $$g_M : Q \to M^{-1}QM$$

   where M is a fixed invertible matrix. Show that g_M is a linear map on V.

3) a) By working out the derivative of

   $$\lim_{n\to\infty} (1 + i\phi/n)^n$$

   with respect to ϕ, show that this expression satisfies the same differential equation as $e^{i\phi}$. You may assume that you can swap the order of the limit and taking the derivative.

   As both functions have the same value at $\phi = 0$ this implies that they are equal by the uniqueness of solutions of ordinary differential equations.

   b) Consider a square matrix A and let $g = e^{iA}$, which is defined via the Taylor series of the exponential. Show that

   $$\lim_{n\to\infty} (\mathbb{1} + iA/n)^n = e^{iA} .$$

Here are some things to ponder:

1. What is a group? Why are symmetries groups?
2. What does the U in $U(1)$ stand for ? What might somebody mean when they say $U(n)$?
3. How many ways to define the exponential function can you come up with ?
4. Tangents are linear approximations.