4 Show using index notation that

a) $(gh)^\dagger = h^\dagger g^\dagger$

b) $\mathrm{tr}(gh) = \mathrm{tr}(hg)$

c) $(g\boldsymbol{v}) \cdot (h\boldsymbol{w}) = \boldsymbol{v}\,(g^T h)\,\boldsymbol{w}$

d) $\det g^\dagger = \overline{\det g}$

where g and h are complex $n \times n$ matrices and $\boldsymbol{v}$ and $\boldsymbol{w}$ are n-dimensional vectors.

5. For a general $k \times k$ matrix M show that

   (a) $\det e^M = e^{\mathrm{tr} M}$.

   (b) Use this to conclude that for $g = e^M$ we have $\log \det g = \mathrm{tr} \log g$. Here the log of a matrix is defined as the inverse function of the exponential.

6. Show that the Pauli matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \ , \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \ , \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{0.1}$$

satisfy

$$[\sigma_i, \sigma_j] = 2i\epsilon_{ijk}\sigma_k \, . \tag{0.2}$$

Here are some things to ponder:

1. What is the group $SU(2)$? What do the S and the U stand for? What might $SU(n)$ be?
2. What are some properties of the Pauli matrices?