7. Let f be a homomorphism between two groups G and H. Show that

   a) $f(e_G) = e_H$ where e_G and e_H are the unit elements of G and H, respectively.

   b) $f(g^{-1}) = f(g)^{-1}$ for any $g \in G$.

8. $U(2)$ is the group of complex 2×2 matrices g such that $g^\dagger = g^{-1}$, with the group composition being matrix multiplication. Let F be the map which sends

$$g \mapsto \det g \,. \tag{0.1}$$

   Show that F is a group homomorphism from $U(2)$ to $U(1)$.

9. (a) Show that

$$\sigma_i \sigma_j + \sigma_j \sigma_i = 2\delta_{ij}\mathbb{1} \tag{0.2}$$

   where σ_i are the Pauli matrices.

   (b) Show that

$$g(\boldsymbol{\alpha}) = e^{i\boldsymbol{\alpha}\boldsymbol{\sigma}} = \begin{pmatrix} \cos(a) + i\sin(a)a_3/a & \sin(a)a_2/a + i\sin(a)a_1/a \\ -\sin(a)a_2/a + i\sin(a)a_1/a & \cos(a) - i\sin(a)a_3/a \end{pmatrix} \tag{0.3}$$

   where $a = \sqrt{\alpha_1^2 + \alpha_2^2 + \alpha_3^2}$. [hint: write $\boldsymbol{\alpha} = a\boldsymbol{n}$ with $|\boldsymbol{n}|^2 = 1$, i.e. $n_j = \alpha_j/a$]

Here are some things you should discuss with your friends:

1. What is a group homomorphism and why is this a good concept?
2. What is the relationship between $SU(2)$ and $SO(3)$?