10. Let G be the set of complex 2×2 matrices of the form

$$g = \begin{pmatrix} \alpha & \beta \\ -\bar{\beta} & \bar{\alpha} \end{pmatrix}$$

for $\alpha, \beta \in \mathbb{C}$ and $|\alpha|^2 + |\beta|^2 \neq 0$.

   a) Show that G is a group using matrix multiplication as the group operation.

   b) Show that $SU(2)$ is a subgroup of G.

   c) Show that $V := \{\gamma | g = e^{i\gamma} \in G\}$ is a vector space and find a basis for V.

11. Which of the following sets are closed? Which are open? For all cases use the standard topology of $\mathbb{R}^n$ or a topology induced from it.

   (a) $\{0 < x < \pi\} \subset \mathbb{R}$ with coordinate x

   (b) $\{x_1 < -2\} \subset \mathbb{R}^2$ with coordinates (x_1, x_2)

   (c) $\{0 < x \leq \pi\} \subset \mathbb{R}$

   (d) $\{0 < x_1 < 1\} \subset \mathbb{R}^2$ with coordinates (x_1, x_2)

   (e) $\mathbb{R}^n \subseteq \mathbb{R}^n$

   (f) $\{(x_1, x_2) \subset \mathbb{R}^2 \,|\, x_1^2 \leq 42 - x_2^2\} \subset \mathbb{R}^2$ with coordinates (x_1, x_2)

   (g) $\{(x_1, x_2) | x_1^2 + x_2^2 = 1\} \subset \mathbb{R}^3$

   (h) $\{(x_1, x_2) | x_1^2 + x_2^2 = 1\} \subset \{(x_1, x_2, x_3) | x_1^2 + x_2^2 + x_3^4 = 1\}$ with the topology induced from $\mathbb{R}^3$

12 Prove that arbitrary unions and finite intersections of open sets in $\mathbb{R}^n$ are again open. Why is the intersection of an infinite number of open sets not open in general ?

13 Consider the sets of points in $\mathbb{R}^2$ with coordinates (x, y) defined implicitely by the following relations

   a) $y = x^3$

   b) $xy = 0$

   c) $x^2 + y^4 = 0$

   d) $x > y$

   e) $y^2 + x^3 - 3x - 2 = 0$

Using the induced topology from $\mathbb{R}^2$, decide in each case if this is a differentiable manifold.
[hint: plot them!]

Here are some things you should discuss with your friends:

1. What is a topology, what is a topological space?
2. What is a manifold?
3. Why do we need to declare which sets we consider open (create a ‘topological space’) before we can define coordinate patches?