14. Consider the stereographic projection of the three-sphere S^3.

    a) Show that the inverse of the map between $\boldsymbol{\varphi}_\pm$ and the x_i is given by

    $$x_1 = \frac{\pm(1 - |\boldsymbol{\varphi}_\pm|^2)}{|\boldsymbol{\varphi}_\pm|^2 + 1}$$
    $$\begin{pmatrix} x_2 \\ x_3 \\ x_4 \end{pmatrix} = \frac{2}{|\boldsymbol{\varphi}_\pm|^2 + 1}\boldsymbol{\varphi}_\pm$$

    b) Consider the coordinate patches defined by stereographic projection on S^3 and find the transition function from U_+ to U_-.

16. $O(1,1)$ are the real 2×2 matrices O which leave the bilinear form $x_1^2 - x_2^2$ invariant when acting on $\boldsymbol{x} = (x_1, x_2)$ as

    $$\boldsymbol{x} \to O\boldsymbol{x}\,.$$

    a) Show that $O(1,1)$ is a group using matrix multiplication.
    b) Find the general form of elements of $O(1,1)$.
    c) Give $O(1,1)$ the structure of a differentiable manifold by equipping it with a suitable topology and write down coordinate charts.
    d) Find the tangent space of $O(1,1)$ at the identity element.

Here are some things you should discuss with your friends:

1. How does stereographic projection work?
2. What is a tangent vector?