17. Show that for every $g \in GL(n,\mathbb{R}) \setminus O(n)$, i.e. $g \in GL(n,\mathbb{R})$ such that $g^T g \neq \mathbb{1}$, there is an open set U_g containing g such that U_g is entirely contained in $GL(n,\mathbb{R}) \setminus O(n)$.

    hint: $GL(n,\mathbb{R})$ inherits its topology from the vector space $V_{n\times n}$ of real $n \times n$ matrices, which is isomorphic to $\mathbb{R}^{n^2}$: the n^2 entries of such a matrix are just the components of a vector in $\mathbb{R}^{n^2}$ from this perspective. We can hence describe the open ball of radius r around a matrix M with components M_{ij} as

    $$B_r(M) = \left\{ N \in V_{n\times n} | \sum_{ij} (N_{ij} - M_{ij})^2 < r \right\} . \tag{0.1}$$

18. $GL(n,\mathbb{C})$ is the group of invertible complex $n \times n$ matrices. Show that $GL(n,\mathbb{C})$ is a Lie group.

19. Find the dimension of the group $SO(n)$ by finding the dimension of its Lie algebra.

20. Consider the set G of matrices

    $$G = \left\{ \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} | a, b, c \in \mathbb{R}, ac \neq 0 \right\} \tag{0.2}$$

    (a) Show that G is a Lie group using matrix multiplication as the group composition.

    (b) Find the Lie algebra $\mathfrak{g}$ of G.

    (c) Compute the exponentials of the basis elements of the Lie algebra you have found.

Here are some things you should discuss with your friends:

1. What is a Lie group?
2. How can you find the Lie algebra of a matrix Lie group?