4. Show using index notation that

   a) $(A+B)^T = A^T + B^T$
   b) $(AB)^T = B^T A^T$
   c) $tr(cA) = c\, tr(A)$
   d) $tr(AB) = tr(BA)$
   e) $tr A^T = tr A$
   f) $tr(A+B) = tr A + tr B$
   g) $(A\boldsymbol{v})\cdot(B\boldsymbol{w}) = \boldsymbol{v}\,(A^T B)\,\boldsymbol{w}$
   h) $\det A^\dagger = \overline{\det A}$
   i) $\det cA = c^n \det A$

   where A and B are complex $n\times n$ matrices, $\boldsymbol{v}$ and $\boldsymbol{w}$ are vectors with n components, and c is a number.

5. For a general $k\times k$ matrix M show that

   a) $\det e^M = e^{trM}$.
   b) Use this to conclude that for $g = e^M$ we have $\log\det g = tr \log g$. Here the log of a matrix is defined as the inverse function of the exponential.

6. Show that the Pauli matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \ , \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \ , \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{0.1}$$

   satisfy

$$[\sigma_i, \sigma_j] = 2i\epsilon_{ijk}\sigma_k \, . \tag{0.2}$$

Here are some things to ponder:

1. What is the group $SU(2)$, what might $SU(n)$ be like?