3) Verify that

$$l^{01}=\begin{pmatrix}0&-1&0&0\\-1&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix}\quad l^{02}=\begin{pmatrix}0&0&-1&0\\0&0&0&0\\-1&0&0&0\\0&0&0&0\end{pmatrix}\quad l^{03}=\begin{pmatrix}0&0&0&-1\\0&0&0&0\\0&0&0&0\\-1&0&0&0\end{pmatrix}$$
$$l^{12}=\begin{pmatrix}0&0&0&0\\0&0&1&0\\0&-1&0&0\\0&0&0&0\end{pmatrix}\quad l^{13}=\begin{pmatrix}0&0&0&0\\0&0&0&1\\0&0&0&0\\0&-1&0&0\end{pmatrix}\quad l^{23}=\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&1\\0&0&-1&0\end{pmatrix} \tag{0.1}$$

are in the Lie algebra of L.

4) The Dirac matrices are

$$\gamma^0=\begin{pmatrix}0&\mathbb{1}_{2\times 2}\\-\mathbb{1}_{2\times 2}&0\end{pmatrix}\,,\qquad \gamma^i=\begin{pmatrix}0&\sigma_i\\\sigma_i&0\end{pmatrix}\quad i=1,2,3 \tag{0.2}$$

where $\mathbb{1}_{2\times 2}$ is the 2×2 identity matrix and σ_i are the Pauli matrices

$$\sigma_1=\begin{pmatrix}0&1\\1&0\end{pmatrix}\,,\quad \sigma_2=\begin{pmatrix}0&-i\\i&0\end{pmatrix}\,,\quad \sigma_3=\begin{pmatrix}1&0\\0&-1\end{pmatrix}\,. \tag{0.3}$$

a) Show that the Dirac matrices obey $\{\gamma^\mu,\gamma^\nu\}=2\eta^{\mu\nu}\mathbb{1}_{4\times 4}$.

b) Show the 'freshers dream':

$$(a_\mu\gamma^\mu)^2=a_\mu a^\mu\mathbb{1}_{4\times 4} \tag{0.4}$$

5) Using the Dirac matrices, show that $S^{\mu\nu}=\frac{1}{4}[\gamma^\mu,\gamma^\nu]$ are equal to

$$S^{0i}=\tfrac{1}{2}\begin{pmatrix}\sigma_i&0\\0&-\sigma_i\end{pmatrix}\,,\qquad S^{jk}=\frac{i}{2}\epsilon_{jkl}\begin{pmatrix}\sigma_l&0\\0&\sigma_l\end{pmatrix} \tag{0.5}$$

where i,j,k only take values $1,2,3$. What does this imply about the reducibility of the representation of $SL(2,\mathbb{C})$ defined by exponentiating the $S^{\mu\nu}$?

Here are some things to ponder:

1. What is the global structure of the Lorentz group?
2. How can we construct a representation of the Lie algebra of L using the Dirac matrices?