

6) Verify that

- a) For an element  $\Lambda(\theta) = e^{l^{12}\theta}$  of the Lorentz group ( $l^{12}$  is one of the generators of the Lorentz algebra introduced in the lectures) show that  $\Lambda(0) = \Lambda(2\pi) = \mathbb{1}$ . Now compare this behavior to the corresponding element of the representation acting on a Dirac spinor:  $\Lambda_{1/2}(\theta) = e^{S^{12}\theta}$ .
- b) Let  $\gamma^5 := i\gamma^0\gamma^1\gamma^2\gamma^3$ . What is  $\frac{1}{2}(\gamma^5 \pm \mathbb{1})\Psi$  for  $\Psi$  a Dirac spinor written in terms of Weyl spinors?

7) How does

$$B^{\mu\nu} \equiv \bar{\Psi}\gamma^\mu\gamma^\nu\Psi$$

transform under Lorentz transformations for  $\Psi$  a Dirac spinor?

8) For a Dirac spinor  $\Psi$  write

$$\bar{\Psi}\gamma^\mu\Psi \quad \text{and} \quad \bar{\Psi}\Psi$$

in terms of Weyl spinors.

9) For a relativistic point particle moving on path  $C$  through space-time, the only Lorentz invariant property of  $C$  is its length. Taking the action of a relativistic particle to be the length of  $C$  and parametrizing  $C$  as  $x^\mu(s)$  we can write this as

$$S[x^\mu, \dot{x}^\mu] = -cm \int_C ds = -cm \int_C \sqrt{-\dot{x}^\mu \dot{x}_\mu} ds. \quad (0.1)$$

for a constant  $m$  and  $c$  the speed of light and  $\dot{x}^\mu = \partial/\partial s x^\mu$ .  $C$  is called the world-line of the particle.

- a) Show that this action is invariant under Lorentz transformations  $x^\mu \rightarrow \Lambda^\mu_\nu x^\nu$ .
- b) Find the equations of motions and show that they are solved by straight lines in space-time.
- c) Set  $s = t$  and expand the action for slow particles to recover the action of a non-relativistic point particle.

Here are some things to ponder:

1. In which ways is the relationship between  $SO(3)$  and  $SU(2)$  the same as the relationship between the Lorentz group and  $SL(2, \mathbb{C})$ .
2. What is a spinor?
3. What is an action?