1. Let G be the set of complex 2×2 matrices of the form

$$g = \begin{pmatrix} \alpha & \beta \\ -\bar{\beta} & \bar{\alpha} \end{pmatrix}$$

for $\alpha, \beta \in \mathbb{C}$ and $|\alpha|^2 + |\beta|^2 \neq 0$.

   a) Show that G is a group using matrix multiplication as the group operation.

   b) Show that $SU(2)$ is a subgroup of G.

   c) Show that $V := \{\gamma | g = e^{i\gamma} \in G\}$ is a vector space (when ignoring the multi-valuedness of the logarithm) and find a basis for V.

2. Which of the following sets are closed in the standard topology of $\mathbb{R}^m$? Which are open?

   (a) $\{0 < x < \pi\} \subset \mathbb{R}$ with coordinate x

   (b) $\{x_1 < -2\} \subset \mathbb{R}^2$ with coordinates (x_1, x_2)

   (c) $\{0 < x \leq \pi\} \subset \mathbb{R}$

   (d) $\{0 < x_1 < 1\} \subset \mathbb{R}^2$ with coordinates (x_1, x_2)

   (e) $\mathbb{R}^n \subseteq \mathbb{R}^n$

   (f) $\{(x_1, x_2) \subset \mathbb{R}^2 \,|\, x_1^2 \leq 42 - x_2^2\} \subset \mathbb{R}^2$ with coordinates (x_1, x_2)

   (g) $\{(x_1, x_2, x_3) | x_1^2 + x_2^2 = 1\} \subset \mathbb{R}^3$

3. Prove that arbitrary unions and finite intersections of open sets in $\mathbb{R}^n$ are again open. Why is the intersection of an infinite number of open sets not open in general ?

Here are some things to ponder:

1. What is the relationship between $SO(3)$ and $SU(2)$?
2. How do open and closed subsets of $\mathbb{R}^m$ work?
3. What properties would you ask of open closed sets without reference to $\mathbb{R}^m$?