1. Consider the sets of points in $\mathbb{R}^2$ with coordinates (x, y) defined implicitely by the following relations

   a) $y = x^3$

   b) $xy = c$

   c) $x^2 + y^4 = 1$

   d) $x > y$

   e) $y^2 + x^3 - 3x - c = 0$

   Using the induced topology from $\mathbb{R}^2$, decide in each case if this is a differentiable manifold.
   [hint: plot them! Note that the word 'differentiable' here refers to the manifold and not the functions I used to define a manifold. The two notions are not unrelated however, details are explained in the non-examinable example 1.10. but this is not needed to answer this question.]

2. Describe the tangent space of $SO(3)$ at the identity.

3. $O(1, 1)$ are the real 2×2 matrices O which leave the bilinear form $x_1^2 - x_2^2$ invariant when acting on $\boldsymbol{x} = (x_1, x_2)$ as

$$\boldsymbol{x} \to O\boldsymbol{x}\,.$$

   a) Show that $O(1, 1)$ is a group using matrix multiplication.

   b) Find the general form of elements of $O(1, 1)$.

   c) Explain why $O(1, 1)$ is a differentiable manifold and write down coordinate charts.

   d) Find the tangent space of $O(1, 1)$ at the identity element.

Here are some things to ponder:

1. Why are homeomorphisms and manifolds defined the way they are?

2. Tangent spaces are linear approximations.