13) Assume that under

$$\begin{aligned} \phi_I &\to \phi_I + \delta_\gamma \phi_I = [(\mathbb{1} + \rho(\gamma))\, \boldsymbol{\phi}]_I \\ \partial_\mu \phi_I &\to \partial_\mu \phi_I + \delta_\gamma \partial_\mu \phi_I = [(\mathbb{1} + \rho(\gamma))\, \partial_\mu \boldsymbol{\phi}]_I \end{aligned} \tag{0.1}$$

$\mathcal{L}$ is not invariant but to linear order in $\delta\phi_I$ and $\delta\partial_\mu\phi_I$ we have

$$\mathcal{L} \to \mathcal{L} + \partial_\mu F^\mu(\phi_I, \partial_\nu \phi_I) \tag{0.2}$$

for some functions $F^\mu(\phi_I, \partial_\nu\phi_I)$.

By following the same steps as done in the proof of theorem 4.4., show that this also leads to a conserved current which you should find.

14) Consider the action

$$S = \int d^4x \bar{\Psi}\left(\gamma^\mu \partial_\mu + m\right)\Psi\,.$$

for a Dirac spinor Ψ.

a) Show that S is Lorentz invariant.
b) Find the conserved charge associated to the $U(1)$ symmetry $\Psi \to e^{i\theta}\Psi$.

15) Consider a field Φ transforming in the adjoint representation of the Lie group $SU(n)$. Show that

$$S = \int d^4x \ \mathrm{tr}\left(\partial_\mu \Phi \partial^\mu \Phi\right)$$

is invariant under the action of $SU(n)$ and find the associated conserved current.

Here are some things to ponder:

1. What does Noether's theorem tell you for a field theory?
2. When do we consider a physical system to be Lorentz invariant?